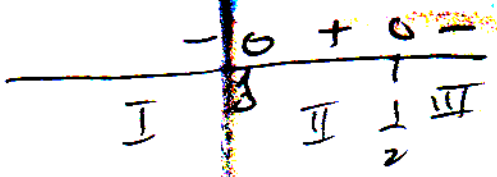
I $\left|\frac{x+1}{x} - 3\right| \le 6$

$\left|\frac{x+1-3x}{x}\right|$

$\left|\frac{1-2x}{x}\right| \le 6$



I $\frac{-(1-2x)}{x} \le 6$

$2x-1 \ge 6x$

$-1 \ge 4x$

$-\frac{1}{4} \ge x$

II $\frac{1-2x}{x} \le 6$

$1-2x \le 6x$

$1 \le 8x$

$\frac{1}{8} \le x$

III $\frac{2x-1}{x} \le 6$

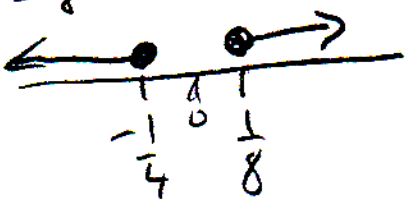
$2x-1 \le 6x$

$-1 \le 4x$

$-\frac{1}{4} \le x$

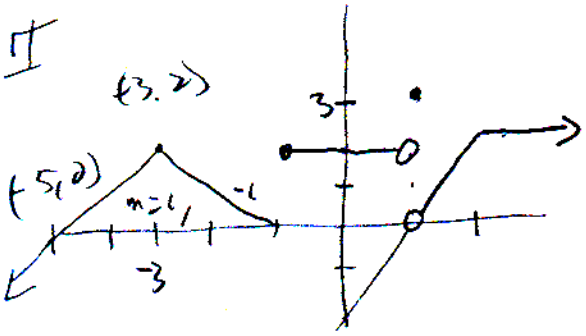
entire 3 intervals!

$x \le -\frac{1}{4}$ or $x \ge \frac{1}{8}$



4 pts

II

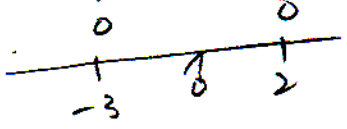


$\frac{2-0}{-3+5} = 1 = m$

$y - 0 = 1(x+5)$

$$f(x) = \begin{cases} x+5 & \text{if } x < -3 \\ -x-1 & \text{if } -3 \le x < -1 \\ 2 & \text{if } -1 \le x < 1 \\ 3 & \text{if } x = 1 \\ 2x-2 & \text{if } 1 < x < 2 \\ 2 & \text{if } x \ge 2 \end{cases}$$

III $A(x) = \left|\,|x-2| + |x+3|\,\right|$



I $|-(x-2) - (x+3)|$

$|-x+2-x-3|$

$|-2x-1|$ since $x < -3$, $-2x > 0$

$-2x-1$

II $|-|x-2| + x + 3|$

$|2+3|$

5

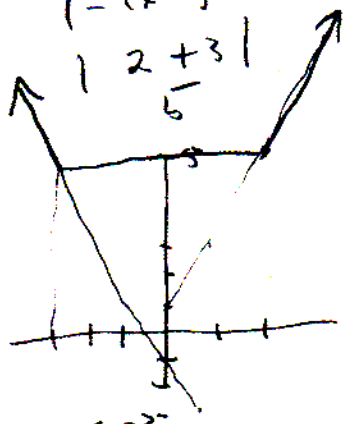
III $|x-2+x+3|$

$|2x+1|$

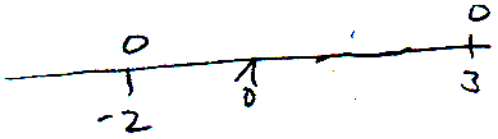
since $x > 2$, $2x+1 > 0$

$2x+1$

$$f(x) = \begin{cases} -2x-1 & \text{if } x < -3 \\ 5 & \text{if } -3 \le x < 2 \\ 2x+1 & \text{if } x \ge 2 \end{cases}$$

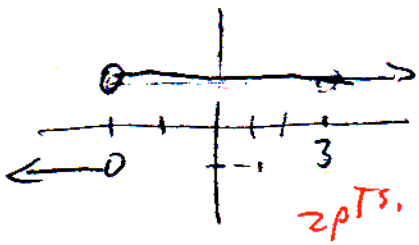


III B $B(x) = (\operatorname{sgn}(x+2))^{\operatorname{sgn}(x-3)}$



$$B(x) = \begin{cases} -1 & \text{if } x < -2 \\ 1 & \text{if } -2 < x < 3 \\ 1 & \text{if } x = 3 \\ 1 & \text{if } x > 3 \end{cases}$$

$(-1)^{-1} = -1$ $(1)^{-1} = 1$ $1^0 = 1$ $1^1 = 1$



NOTE $x \neq -2$ because you would have $0^{-1} = \frac{1}{0}$

2 pts.

IV

$$f(x)=\sqrt{\frac{(2+x)(3-x)}{6+x-x^2}}$$

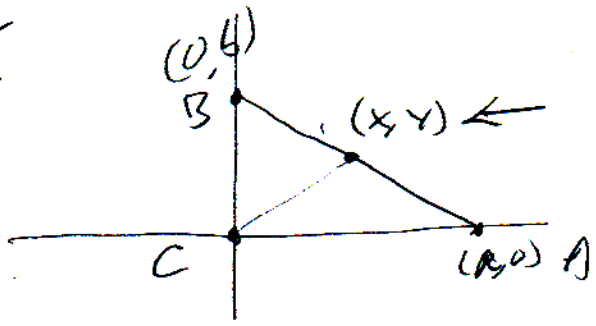
$$\frac{2x+1}{}$$

$+ \;0\; + \;0\; -$ over $-2,\ 3,\ 5$

$- \;0\; +$ over $-\frac{1}{2},\ 3$

D_f $\quad x<-2$ or $-\frac{1}{2}<x\leq 3$

V



hyp dist $=\sqrt{a^2+b^2}$ midpt yields $\frac{\sqrt{a^2+b^2}}{2}$

$(x,y) \leftarrow \left(\frac{a}{2},\frac{b}{2}\right)$

$\sqrt{\left(a-\frac{a}{2}\right)^2+\left(\frac{b}{2}-0\right)^2}$ midpt to A

$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}$

$\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}=\frac{\sqrt{a^2+b^2}}{2}$

$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}-b\right)^2}$

$\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$?

$\frac{\sqrt{a^2+b^2}}{2}=$ midpt to B, $\quad \frac{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}}{}$

$\frac{\sqrt{a^2+b^2}}{2}$ midpt to C. all $=\frac{\sqrt{a^2+b^2}}{2}$

$\therefore$ equidistant.

VI

$f(x)=\frac{3-2x}{5x+2}$

extrema

$f^{-1}(x)$ $\quad x=\frac{3-2y}{5y+2}$

$5xy+2x=3-2y$

$(5x+2)y=3-2x$

$y=\frac{3-2x}{5x+2}$

$$f^{-1}(f(x))=\frac{3-2\left(\frac{3-2x}{5x+2}\right)}{5\left(\frac{3-2x}{5x+2}\right)+2}$$

$\frac{15x+6-6+4x}{5x+2}$

$\frac{15-10x+10x+4}{5x+2}$

$\frac{19x}{19}=x$

$24=10\sin\left(3\theta-\frac{\pi}{2}\right)+4$

$y=5\left(\sin\left(3\theta-\frac{\pi}{2}\right)+2\right)$

D_y $\quad \theta\in\mathbb{R}$

R_y $\quad -5+2=-3$

$5+2=7$

$-3\leq y\leq 7$

2/9/00

	4	5	7
100	0	0	0
90	1	1	
80	3	2	
70	9	9	
60	5	7	
59	3	3	

1452 71.3 70.3
4 71.2
~~69.7~~
71.1

Students were to compute the grade they thought they should receive but came to class unprepared etc. Talked too much too much Joking etc.

No changes in exam.

Honors Analysis

Name AnL Ch 1

Per _____ Date 2000

Copy original problem.

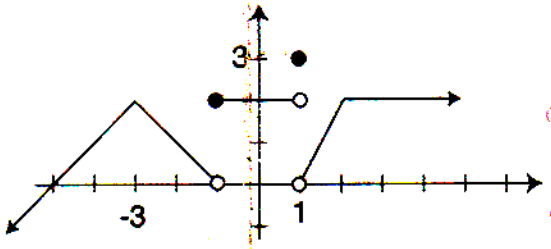
Convince *me* that **you** understand the concept!

No Calculators!

Chapter 1 Exam

I Solve and graph solution on a number line: $\left|\frac{x+1}{x}-3\right| \le 6$ (15 pts)

II Given the graph of the piecewise function, write the function itself (20 pts)



III Rewrite the function as a piecewise function and sketch the graph. (10 pts ea)

a) $A(x) = \big||x-2|+|x+3|\big|$

b) $B(x) = (\operatorname{sgn}(x+2))^{\operatorname{sgn}(x-3)}$

IV Given the function, state the Domain of the function: $f(x) = \sqrt{\frac{\operatorname{sgn}(6+x-x^2)}{2x+1}}$ (15 pts tot)

V The coordinates of the verticies of a right triangle are: $P_1(0,0)$, $P_2(a,0)$, and $P_3(0,b)$. (15 pts)
Show that the midpoint of the hypotenuse is equidistant from all three verticies.
Be *very* clear.

VI Given $f(x)$, *prove* f is a 1 : 1 function: $f(x) = \frac{3-2x}{5x+2}$ (15 pts)

Extra Credit ---------- 5 pts ----------

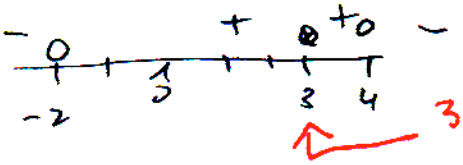
What is the Domain and Range of the function y given: $2y = 10\sin\left(3\theta - \frac{\pi}{6}\right) + 4$

I

$A(x) = \sqrt{\dfrac{(2+x)(3-x)}{x^2-2x+12}}$

$x^2-2x+12$

$(x-3)(x-4)$



3 pts.

$D_A \quad -2 \le x < 4;\ x \neq 3$

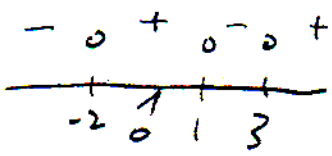
$B(x) = \sqrt{\dfrac{3}{5x+10} + \dfrac{2}{5x-15}}$

$5(x+2) \quad 5(x-3)$

$\sqrt{\dfrac{3x-9+2x+4}{5(x+2)(x-3)}}$

$\sqrt{\dfrac{5x-5}{5(x+2)(x-3)}}$

$= \sqrt{\dfrac{x-1}{(x+2)(x-3)}}$



$D_B \quad -2 < x \le 1$ or $x > 3$

10 still not simplified etc -4

Not factored -6

C) $c(x) = \dfrac{1}{1 - \operatorname{Sgn}(3x+7)}$ $\operatorname{Sgn} x \neq 1$ $\therefore x \le 0$

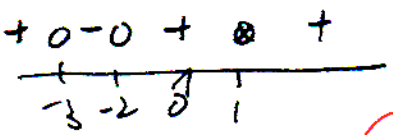
2 pts

$3x+7 \overset{\text{ST}}{\ge} 0 \quad \therefore D_C \quad x \ge -\frac{7}{3}$

$3x \ge -7$

$x \ge -\frac{7}{3}$

D) $D(x) = \sqrt{\dfrac{(x+3)^3 \operatorname{Sgn}(x+2)}{(x-1)^2}}$

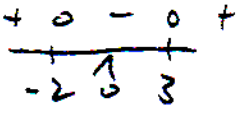


$D_D \quad x \le -3$ or $x \ge -2$; $x \neq 1$.

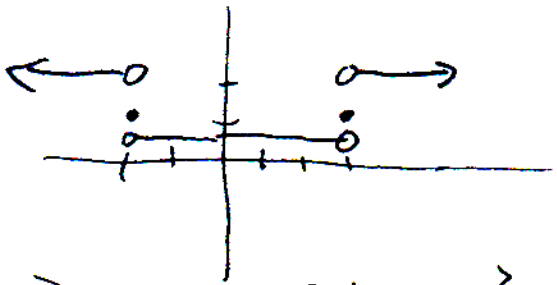
2 pts

II

a) $A(x) = 2^{\operatorname{Sgn}((x+1)(x-3))}$

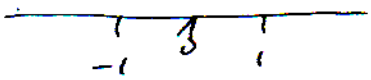


$$A(x) = \begin{cases} 2 & \text{if } x < -2 \\ 1 & \text{if } x = -2 \\ \frac{1}{2} & \text{if } -2 < x < 3 \\ 1 & \text{if } x = 3 \\ 2 & \text{if } x > 3 \end{cases}$$



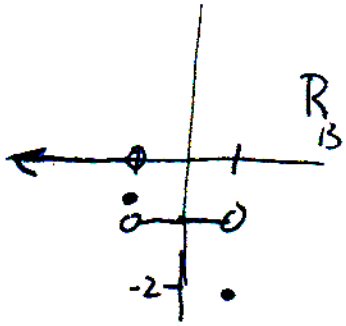
$R_A \quad A(x) \in \{\frac{1}{2}, 1, 2\}$.

$B(x) = \dfrac{(\operatorname{Sgn}(x+1)+1)}{(\operatorname{Sgn}(x-4)-1)}$



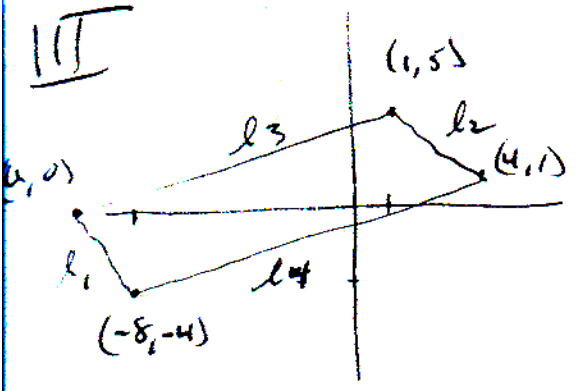
$$B(x) = \begin{cases} 0 & \text{if } x < -1 \\ -\frac{1}{2} & \text{if } x = -1 \\ -1 & \text{if } -1 < x < 1 \\ -2 & \text{if } x = 1 \end{cases}$$

$\frac{1}{-2}$ $\frac{2}{-2}$ $\frac{2}{-1}$



$R_B \quad B(x) \in \{-2, -1, -\frac{1}{2}, 0\}$

III



$m_1 = m_2$ and $m_3 = m_4$ ← must mention

$\frac{5-1}{1-4} \overset{\text{Set}}{=} \frac{0+4}{a+8}$ $\quad\quad$ $\frac{5-0}{1-a} \overset{\text{Set}}{=} \frac{1+4}{4+8}$

$-\frac{4}{3} = \frac{4}{a+8}$ $\quad\quad$ $\frac{5}{1-a} = \frac{5}{12}$

$-4a - 32 = 12$ $\quad\quad$ $12 = 1-a$

$-4a = 44$ $\quad\quad$ $a = -11$

$a = -1$ $\iff$

$\therefore a = -11$ opposite sides ||.

10 pts

IV

$f(x) = \frac{2x+3}{5x-1}$ $\quad$ $g(x) = \frac{2x+3}{7-x}$

a) $x = \frac{2z+3}{5z-1}$

$5xz - x = 2z + 3$

$(5x-2)z = x+3$

$z = \frac{x+3}{5x-2} = f^{-1}(x)$

b) $f\left(\frac{x+3}{5x-2}\right) = \frac{2\left(\frac{x+3}{5x-2}\right)+3}{5\left(\frac{x+3}{5x-2}\right)-1}$

$= \frac{\frac{2x+6+15x-6}{5x-2}}{\frac{5x+15-5x+2}{5x-2}}$

$\frac{17x}{17}$

x

c) $f^{-1}\left(\frac{2x+3}{5x-1}\right) =$

$\frac{\frac{2x+3}{5x-1}+3}{5\left(\frac{2x+3}{5x-1}\right)-2}$

$\frac{\frac{2x+3+15x-3}{5x-1}}{\frac{10x+15-6x+2}{5x-1}}$

$\frac{17x}{17}$

x

d) $D(x) = \sqrt{\frac{1}{f} + \frac{1}{g}}$

$= \sqrt{\frac{5x-1}{2x+3} + \frac{7-x}{2x+3}}$

$= \sqrt{\frac{4x+6}{2x+3}}$

$= \sqrt{2}$ not simplified 2 pts

e) D_0 $x \neq \left\{-\frac{3}{2}, \frac{1}{5}, 7\right\}$

No Additional restrictions.

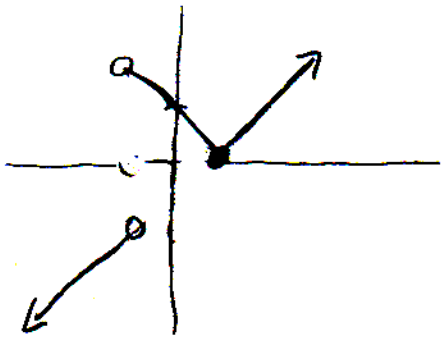
$x \neq \frac{-3}{2}$ ← -4 pts

Extra.

$f(x) = \frac{|x-1|}{\text{sgn}(x+1)}$

	0	−	0	+
		↓	↑	
		−1	1	

$f(x) = \begin{cases} x-1 & \text{if } x < -1 \\ -x+1 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x \geq 1 \end{cases}$ $\quad$ $\frac{-(x-1)}{-1}$ $\quad$ $-(x-1)$



Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

Name Analysis Ch 1

Per _____ Date 2/7/2001

Chapter 1 Exam

I State the domain for each of the following: (30 pts tot)

a) $A(x)=\sqrt{\dfrac{(2+x)(3-x)}{x^2-7x+12}}$

b) $B(x)=\sqrt{\dfrac{3}{5x+10}+\dfrac{2}{5x-15}}$

c) $C(x)=\dfrac{1}{1-\operatorname{sgn}(3x+7)}$

d) $D(x)=\sqrt{\dfrac{(x+3)^3\operatorname{sgn}(x+2)}{(x-1)^2}}$

II For each of the following, re-define as a "piece-wise" function. Sketch a graph. Specifically state the range of the function. (10 pts ea)

a) $A(x)=2^{\operatorname{sgn}((x+2)(x-3))}$

b) $R(x)=\dfrac{(\operatorname{sgn}(x+1)+1)}{(\operatorname{sgn}(x-1)-1)}$

III Given coordinates: $P(1,5)$, $Q(4,1)$, $R(-8,-4)$, and $S(a,0)$. (that is, a is on the x-axis) (25 pts)

give point S

The figure, $PQRS$ is formed by ***connecting the points in order.*** Determine a so the figure formed is a parallelogram. Be convincing that you have a parallelogram (and not a trapazoid, for example). Explain.

IV Given $f(x)=\dfrac{2x+3}{5x-1}$ and $g(x)=\dfrac{2x+3}{7-x}$, and $h(x)=\sqrt{\dfrac{1}{f(x)}+\dfrac{1}{g(x)}}$. (25 pts tot)

a) Find $f^{-1}(x)$

b) Find $f\left(f^{-1}(x)\right)$

c) Find $f^{-1}(f(x))$

d) Find the formula for $h(x)$.

e) Find the domain for $h(x)$.

Extra Credit ---------- 5 pts ----------

Draw the graph of: $F(x)=\dfrac{|x-1|}{\operatorname{sgn}(x+1)}$

	4	5	T
100	0	0	0
90	2	5	7
80	5	9	14
70	7	5	12
60	3	4	7
59	2	2	4

99

8

1381	1930	334
18	24	42
76.7	80.4	78.8

an excellent exam.
It was shortened version of
1998 exam. Because
of "shortened Wednesday"
change as indicated on
opposite side.

9F VC
89 JG

I

$A(x) = \sqrt{9 - x^2}$ — $-$ 0 $+$ 0 $-$ on $-3, 0, 3$

Not factored → $(3-x)(3+x)$ → −2

or $D_A \; -3 \le x \le 3$

No number line −3

$B(x) = \sqrt{1 + \frac{1}{x-1}} = \sqrt{\frac{x-1+1}{x-1}} = \sqrt{\frac{x}{x-1}}$

$+ \; 0 \; - \; \| \; +$ on $0, 1$

$D_B \; x \le 0$ or $x > 1$

Not single fraction −4

$C(x) = \sqrt{25 - x^2}$ ±5 −3 pts

$D_C \; x \in \mathbb{R}$

$D(x) = \sqrt{x^2 - x - 2}$

$(x-2)(x+1)$

$+ \; 0 \; - \; 0 \; +$ on $-1, 0, 2$

$D_D \; x \le -1$ or $x \ge 2$

II

$A(x) = \operatorname{Sgn}(x-1) - |x+1|$

0 0 on $-1, 1$

$$A(x) = \begin{cases} x & ; \; x < -1 \\ -x-2 & ; \; -1 \le x < 1 \\ -2 & ; \; x = 1 \\ -x & ; \; x > 1 \end{cases}$$

$-1 + x + 1 = x$

$-1 - x - 1$

$0 \quad -2$

$1 \quad -x-1$

$R_A \; A(x) \le -1$

$B(x) = x + |x| + x \operatorname{Sgn} x$

$$B(x) = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \end{cases}$$

$R_B \; B(x) \ge 0$

$D_B \; x \le 0$ or $x > 1$

D_D 0

$x - x - x = -x$

$x + x + x$

III

$f(x) = \frac{x+4}{x^2+x-12} = \frac{x+4}{(x+4)(x-3)} = \frac{1}{x-3}$

$D_f \; x \ne \{-4, 3\}$

$g(x) = \frac{(4-x)^2}{(x-3)(x+2)}$

$D_g \; x \ne \{3, -2\}$

III A

$A(x) = \sqrt{\frac{1}{g(x)}}$

$+ \; 0 \; - \; \| \; + \; 0 \; +$ on $-2, 0, 3, 4$ — 3 pts

$D_A \; x < -2$ or $x > 3$, $x \ne 4$

III B

3) $B(x) = f(f(x))$

$\frac{1}{x-3} \overset{\text{set}}{=} -4 \qquad \frac{1}{x-3} \overset{\text{set}}{=} 3$

$1 = -4x + 12 \qquad 1 = 3x - 9$

$4x = 11 \qquad 3x = 10$

$x = \frac{11}{4} \qquad x = \frac{10}{3}$

$D_B \; x \ne \left\{-4, 3, \frac{11}{4}, \frac{10}{3}\right\}$

III C

C) $C(x) = \sqrt{f(x) - g(x)}$

$- \; 0 \; + \; 0 \; - \; \| \; + \; 0 \; -$ on $-2, 0, 2, 3, 7$

$D_C \; -2 < x \le 2$ or $3 < x \le 7$

$\frac{1}{x-3} - \frac{(4-x)^2}{(x-3)(x+2)}$

$\frac{x+2 - (16 - 8x + x^2)}{(x-3)(x+2)}$

$\frac{x + 2 - 16 + 8x - x^2}{(x-3)(x+2)}$

$\frac{-14 + 9x - x^2}{(\quad)(\quad)}$

$-\frac{(x^2 - 9x + 14)}{(\quad)(\quad)}$

$-\frac{(x-7)(x-2)}{(x-3)(x+2)}$

Not combined. −6

IV $f(x)=\frac{2x+1}{x-2}$ $g(x)=\frac{1}{x+1}$ $h(x)=f(g(x))$

a) $h(x)=f\left(\frac{1}{x+1}\right)=\frac{2\left(\frac{1}{x+1}\right)+1}{\frac{1}{x+1}-2}$

$=\frac{\frac{2+x+1}{x+1}}{\frac{1-2x-2}{x+1}}$

$=\frac{x+3}{-2x-1}$

$=-\frac{x+3}{2x+1}$

b) $h(x)=-\frac{(x+3)}{2x+1}$ $f^{-1}(f(x))=$

$x=-\frac{(z+3)}{(2z+1)}$

$-2zx-x=z+3$

$-2zx-z=x+3$

$-z(2x+1)=$

$-z=\frac{x+3}{2x+1}$

$f^{-1}(x)\ z=-\frac{x+3}{2x+1}$

$f'(f(x))=$

$f'\left(-\frac{x+3}{2x+1}\right)=$

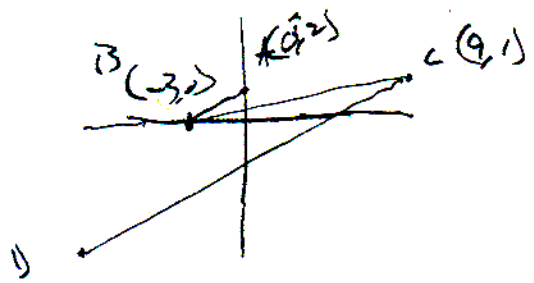
$=-\frac{-\frac{x+3}{2x+1}+3}{2\left(\frac{-x-3}{2x+1}\right)+1}$

$=-\frac{\frac{-x-3+6x+3}{2x+1}}{\frac{-2x-6+2x+1}{2x+1}}$

$=-\frac{5x}{-5}$

$=x$

V A(0,2) B(−3,0) C(9,1) D(−6,−9)



a) $m(\overline{AB})=\frac{2-0}{0+3}=\frac{2}{3}=m_1$

$m(\overline{CD})=\frac{1+9}{9+6}=\frac{10}{15}=\frac{2}{3}=m_2$

$\therefore m_1=m_2$ Reason. Lines are ∥. conclusion

b) $d=\sqrt{1^2+12^2}$

$d=\sqrt{145}$ length fm B to C.

E

$\frac{\text{miles}}{\text{hr}}\cdot\frac{1\text{ hr}}{60\text{ min}}\cdot\frac{1\text{ min}}{60\text{ sec}}$

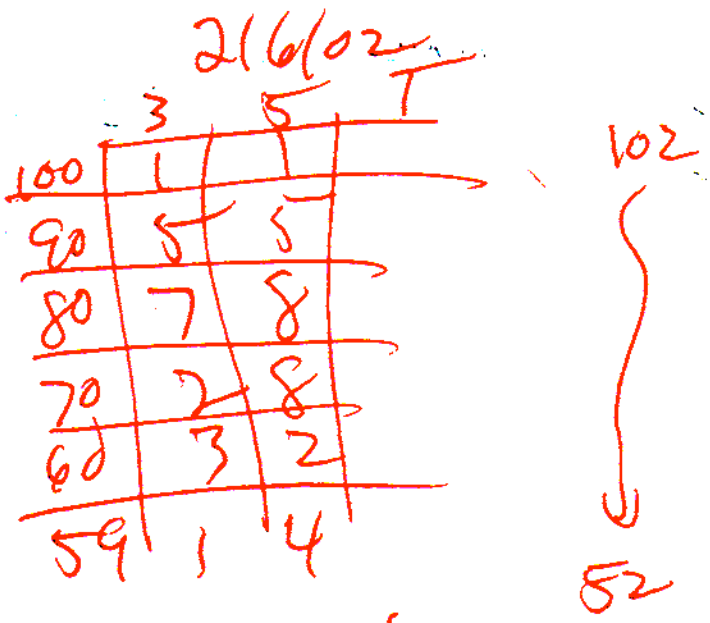
$\frac{1}{36}\ \frac{\text{mile}}{\text{sec}}$.

3 sec to enter tunnel no $3\cdot\frac{1}{36}=\frac{1}{12}$

Train is $\frac{1}{12}$ mile long. $=440$ ft.

30 sec for last cart to exit tunnel

$30\cdot\frac{1}{36}=\frac{5}{6}$ Train is $\frac{5}{6}$ mile long, $=4400$ ft.



1881/19 2188/28 3769/47

83.2 78.14 80.2

Quite a few sleepy, lazy students not bothering to factor etc. Using examinefortun.

Excellent, rather easy, exam.

Honors Analysis
Copy original problem.

Name AnL Ch 1
Per _____ Date 2002

Convince *me* that **you** understand the concept!

No Calculators.

Chapter 1 Exam

I

Determine the domain for each of the following functions. (5 pts ea)

a) $A(x) = \sqrt{9 - x^2}$ b) $B(x) = \sqrt{1 + \frac{1}{x-1}}$ c) $C(x) = \sqrt{25}$ d) $D(x) = \sqrt{x^2 - x - 2}$

II

For each of the following, re-define as a "piece-wise" function. Sketch the graph. Specifically state the range of the function. (10 pts ea)

a) $A(x) = \text{sgn}(x-1) - |x+1|$ b) $B(x) = x + |x| + x\,\text{sgn}(x)$

III

Given $f(x) = \frac{x+4}{x^2+x-12}$ and $g(x) = \frac{(4-x)^2}{(x-3)(x+2)}$. Determine the domain for each of the following (~~Do not actually compute the formula for the new functions.~~): (10 pts ea)

Final

a) $A(x) = \sqrt{\frac{1}{g(x)}}$ b) $B(x) = f(f(x))$ c) $C(x) = \sqrt{f(x) - g(x)}$

IV

Given $f(x) = \frac{2x+1}{x-2}$, $g(x) = \frac{1}{x+1}$ and $h(x) = f(g(x))$ (20 pts tot)

a) Determine the formula for $h(x)$.

b) Prove $h(x)$ is a 1 : 1 funnction.

V

Given: $A(0,2)$, $B(-3,0)$, $C(9,1)$, $D(-6,-9)$, (5 pts ea)

a) Prove the line segment $\overline{AB}$ is parallel to the line segment $\overline{CD}$.

b) Find the length of the line segement $\overline{BC}$

Extra Credit ---------- 5 pts ----------

Final

A train traveling at 100 miles per hour takes three seconds to enter a tunnel and an additional thirty seconds to pass completely through the tunnel.

1. What is the length of the train?
2. How long is the tunnel?

(Is it true that the more math you know, the harder it is to do arithmetic?)

2.4.4.03

100	0	0	0
90	3	1	4
80	7	3	10
70	5	4	9
60	4	8	12
54	3	7	10

99

25

1726/22 = 78.5

1499/23 = 65.2

3225/45 = 71.7

Great exam.
Lazy students.
Most missed #4 Lazy
#5 the problem
Extra D20 to get it.

Honors Analysis

CakeWalk Algebra

I Draw number lines and indicate positive or negative as appropriate. Also specifically state positive intervals.

A) $(x+1)$ B) $(x+1)(x-2)$ C) $(x+1)^2(x-2)$ D) $\dfrac{(x-1)(x+3)}{(x-3)}$

E) $\dfrac{(x-1)^2(x+2)}{(x-3)}$ F) $\dfrac{(x-1)(x+2)}{(x+3)(x-3)}$ G) $\dfrac{(x-1)(x+2)}{(x+3)(x-3)^2}$ H) $\dfrac{(x-1)(x+2)}{(x-3)(x-2)^2}$

I) $(x-1)^2$ J) $\dfrac{(x+4)(x-1)}{(x-3)(x-4)}$ K) x^3+1 L) x^4-16

II State the DOMAIN *only*. Graph domain on a number line.

A) $A(x)=\sqrt{\dfrac{x^2-5x+6}{x+1}}$ B) $B(x)=\dfrac{\sqrt{x^2-5x+6}}{x+1}$ C) $C(x)=\dfrac{\sqrt{x-2}}{x^2+x}$

D) $D(x)=\sqrt{\dfrac{x+2}{x-1}}$ E) $E(x)=\sqrt{-x}$ F) $F(x)=\sqrt{9-x^2}$

G) $G(x)=\dfrac{x\sqrt{x}}{x^2-5x+6}$ H) $H(x)=\sqrt{\dfrac{x+3}{x^2+x}}$ I) $H(x)=\dfrac{\sqrt{x+3}}{x^2+x}$

III Write each of the following as a "piece-meal function". Graph the function. Specifically state the range.

A) $A(x)=2|x-1|+1$ B) $B(x)=|x+1|\,\mathrm{Sgn}(x-1)$ C) $C(x)=x+x\,\mathrm{Sgn}(x-2)$

D) $D(x)=[\mathrm{Sgn}(x-1)]^2+\mathrm{Sgn}(x+2)$ E) $E(x)=\mathrm{Sgn}(x-2)+|x+2|$ F) $F(x)=|x+|x||$

G) $G(x)=[\mathrm{Sgn}(x+2)][\mathrm{Sgn}(x+2)]$ H) $H(x)=\dfrac{|2x|}{x}$

IV State the DOMAIN *ONLY*. Do NOT compute the composition!

Given: $P(x)=\dfrac{x-3}{x^2-6x+9}$ and $Q(x)=\dfrac{x+3}{x^2+3x+2}$

A) $A(x)=\dfrac{P(x)}{Q(x)}$ B) $B(x)=Q(P(x))$ C) $C(x)=P(P(x))$

D) $D(x)=\sqrt{P(x)}$ E) $E(x)=\dfrac{1}{P(x)+Q(x)}$ F) $F(x)=\dfrac{Q(x)}{P(x)}$

G) $G(x)=\sqrt{\dfrac{1}{Q(x)}}$ H) $H(x)=Q(Q(x))$ I) $I(x)=\dfrac{\sqrt{P(x)}}{Q(x)}$

"Obviously.....

these are oral exercises.... but we will write the solutions out anyway!"

I Find $f(2+h)$, $f(x+h)$, and $\dfrac{f(x+h)-f(x)}{h}$ (where $h \neq 0$) for

a) $f(x) = x - x^2$ b) $f(x) = \dfrac{x}{x+1}$

II Find the domain of ...

a) $f(x) = \dfrac{x+2}{x^2-1}$ b) $f(x) = \dfrac{x^4}{x^2+x-6}$ c) $f(x) = \sqrt[4]{x^2-6x}$

d) $f(x) = \sqrt[4]{7-3x}$ e) $f(x) = \sqrt[3]{x-1}$

III Find the domain and range and sketch the graph of the function $f(x) = \sqrt{4-x^2}$.

IV Find the domain and sketch the graph of the function:

a) $f(x) = 3-2x$ b) $f(x) = x^2+2x-1$ c) $f(x) = \sqrt{x-5}$

d) $f(x) = \sqrt{6-2x}$ e) $f(x) = x + |x|$ f) $f(x) = |2x|$

g) $f(x) = \dfrac{x}{|x|}$ h) $f(x) = \dfrac{x^2+5x+6}{x+2}$ i) $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x+1 & \text{if } x > 0 \end{cases}$

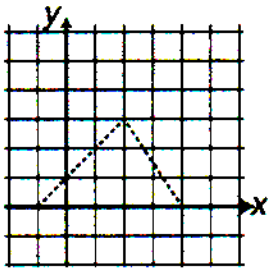
j) $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x > +1 \end{cases}$ k) $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$ l) $f(x) = \begin{cases} -1 & \text{if } x < 1 \\ 3x+2 & \text{if } |x| < 1 \\ 7-2x & \text{if } x > 1 \end{cases}$

m) $f(x) = \sqrt{9-x^2}$ n) $f(x) = \sqrt{x^2-5x-6}$ o) $f(x) = \sqrt{x^2-5x+6}$

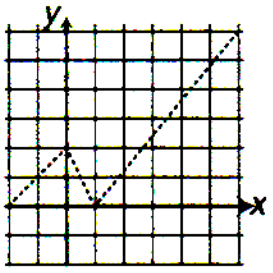
V Find an expression for the function whose graph is the given curve:

a) The line segment joining the points $(-2, 1)$ and $(4, -6)$.
b) The line segment joining the points $(-3, -2)$ and $(6, 3)$
c) The bottom half of the parabola $x + (y-1)^2 = 0$
d) The top half of the circle $(x-1)^2 + y^2 = 1$

e)



f)



Honors Analysis
Chapter 1
2003

Analysis is the bridge between algebra and trigonometry to calculus. To be effective, we all need to have the same definitions for the same words, the same methods and procedures for the same situations, and the willingness to try *something* even when the solution is not obvious.

# 1 R/C Pg 1 - 11 DO # 2 Q & A to oral ex Pg 4-5 and Pg 9-10 ▪▪▪▪▪▪▪▪▪▪▪▪▪▪▪ # 3 Section I "CakeWalk Algebra" (on back of HW sked) 1 / 27	# 4 Pg 5 13 - 18 Pg 10 12 - 19 Pg 10 20 - 25 State Domain ONLY **and** CakeWalk II 1/28	# 5 Pg 11 26 - 40 replace text instructions with: "Substitute and Simplify" ▪▪▪▪▪▪▪▪▪▪▪▪▪▪▪ #6 R/C Pg 12 - 15 INCLUDE Q & A to Oral ex Pg 16 1 / 29	# 7 Pg 16 - 17 1 - 6, 9, 17 - 26 "normal" = perpendicular For 17-20 you are seeking the point (x,y). We don't do "guess and check" since we are not mathematically challenged! 1/ 30	# 8 "Explain **completely** *EVERYTHING there is to know*" about the 5 functions on Pg 20 and 21. **Also** # 9 Write answers to Oral exercises Pg 23 pblms 18 through 25 and 32 - 37 1 / 31
# 10 Pg 24 - 25 23 - 32 **Create piece-meal fctns for 27 – 32**	# 11 CakeWalk III 2 / 4	# 12 Pgs 37 - 38 1 - 20 find G(F(x)) only 21 - 26 find domain-ONLY and CakeWalk IV	# 13 Pg 45 - 46 1 - 23 odd ; 25 - 27 Pblm 27 is very important! 2 / 6	# 14 Pg 45 - 46 2 - 24 even; 28 - 30 Pblm 30 is very important! ▪▪▪▪▪▪▪▪▪▪▪▪▪▪▪ # 15 Pg 49 - 50 4, 5, 6, 8 , 9 , 10 On Pblm 6, **also** make "piece-meal" fctn 2 / 7
School Holiday $\left(f \circ f^{-1}\right)(x)$ means $f\left(f^{-1}(x)\right)$ 2 / 10	***MAA Exam!!!!*** # 16 ,17 **"Obviously ..."**	2 / 12	Put last year's exam on the board. Come to class prepared! 2 / 13	**EXAM CHAPTER 1** 2 / 14

I A $f(x)=\frac{1}{x+1}$ D_f $\mathbb{R}\ x\neq -1$ R_f $\mathbb{R}\ f(x)\neq 0$

$g(x)=\frac{x}{x+1}$ D_g $\mathbb{R}\ x\neq -1$ R_g $\mathbb{R}\ g(x)\neq 1$

$\frac{f(x)}{g(x)} = \frac{\frac{1}{x+1}}{\frac{x}{x+1}} = \frac{1}{x}$ $D_{\frac{f}{g}}$ $\mathbb{R}\ x\neq 0,-1$

$f(g(x)) = f\left(\frac{x}{x+1}\right) = \frac{1}{\frac{x}{x+1}+1} = \frac{1}{\frac{x+x+1}{x+1}} = \frac{x+1}{2x+1}$ $D_{f\circ g} \Rightarrow x\in\mathbb{R}\ x\neq -1,-\frac{1}{2}$

$g(f(x)) = g\left(\frac{1}{x+1}\right) = \frac{\frac{1}{x+1}}{\frac{1}{x+1}+1} = \frac{\frac{1}{x+1}}{\frac{1+x+1}{x+1}} = \frac{1}{x+2}$ $D_{g\circ f} \Rightarrow x\in\mathbb{R}\ x\neq -1,-2$

$y=\frac{1}{x+1}$, $x=\frac{1}{y+1}$, $xy+x=1$, $xy=1-x$, $y=\frac{1-x}{x}$

$f^{-1}(x)=\frac{1-x}{x}$ $D_{f^{-1}} \Rightarrow x\neq 0\ x\in\mathbb{R}$

$y=\frac{x}{x+1}$, $x=\frac{y}{y+1}$, $xy+x=y$, $xy-y=-x$, $y(x-1)=-x$

$y=\frac{-x}{x-1}=\frac{x}{1-x}$ $g^{-1}(x)=\frac{x}{1-x}$ $D_{g^{-1}}$ $x\in\mathbb{R}\ x\neq 1$

$D_{f\circ g}$ $x\neq -1$ ⊕ $\frac{x}{x+1}\neq -1 \Rightarrow x\neq -x-1$, $2x\neq -1$, $x\neq -\frac{1}{2}$

$\therefore D_{f\circ g} \Rightarrow x\in\mathbb{R}, x\neq -1, -\frac{1}{2}$.

$D_{g\circ f}$ $x\neq -1$ ⊕ $\frac{1}{x+1}\neq -1$, $1\neq -x-1$, $x\neq -2$ $\therefore$ $x\neq -1,-2$.

1:1? $f(f^{-1})\stackrel{?}{=}x$

$f\left(\frac{1-x}{x}\right)=\frac{1}{\frac{1-x}{x}+1}=\frac{1}{\frac{1-x+x}{x}}=\frac{1}{\frac{1}{x}}=x$ Yes, f is 1:1.

I B $f(x)=\frac{2x+1}{3x-2}$ D_f $x\in\mathbb{R}$ $x\neq\frac{2}{3}$ R_f $f(x)\in\mathbb{R}$ $f(x)\neq\frac{2}{3}$

$g(x)=\frac{x-1}{x-2}$ D_g $x\in\mathbb{R}$ $x\neq 2$ R_g $g(x)\in\mathbb{R}$ $g(x)\neq 1$

$\frac{f(x)}{g(x)} = \frac{\frac{2x+1}{3x-2}}{\frac{x-1}{x-2}} = \frac{(2x+1)}{(3x-2)}\cdot\frac{x-2}{x-1} = \frac{2x^2-3x-2}{3x^2-5x+2}$ $D_{\frac{f}{g}}$ $x\in\mathbb{R}\ x\neq\{\frac{2}{3},1,2\}$

$\frac{x-1}{x-2}\stackrel{?}{=}0$ only if $x=1$ $\therefore$

$f^{-1}(x) \Rightarrow y=\frac{2x+1}{3x-2}$, $x=\frac{2y+1}{3y-2}$, $3xy-2x=2y+1$, $3xy-2y=2x+1$,

$y(3x-2)=2x+1$, $y=\frac{2x+1}{3x-2}$ $D_{f^{-1}}$ $x\in\mathbb{R}\ x\neq\frac{2}{3}$.

$f^{-1}(x)=\frac{2x+1}{3x-2}$.

$g^{-1}(x) \Rightarrow y=\frac{x-1}{x-2}$, $x=\frac{y-1}{y-2}$, $xy-2x=y-1$, $xy-y=2x-1$,

$y(x-1)=2x-1$, $y=\frac{2x-1}{x-1}$, $g^{-1}(x)=\frac{2x-1}{x-1}$ $D_{g^{-1}}$ $x\in\mathbb{R}\ x\neq 1$

I b cont.

$f(g(x)) = f\left(\frac{x-1}{x-2}\right) = \frac{2\left(\frac{x-1}{x-2}\right)+1}{3\left(\frac{x-1}{x-2}\right)-2} = \frac{\frac{2x-2+x-2}{x-2}}{\frac{3x-3-2x+4}{x-2}} = \frac{\frac{3x-4}{x-2}}{\frac{x+1}{x-2}} = \frac{3x-4}{x+1}$

$D_{f\circ g}$. $g \Rightarrow x \neq 2$ $\quad f \Rightarrow x \neq \frac{2}{3} \Rightarrow \frac{x-1}{x-2} \neq \frac{2}{3}$, $\quad 3x-3 \neq 2x-4$, $\quad x \neq -1$

$D_{f\circ g}$ $x \in \mathbb{R}$, $x \notin \{-1, 2\}$.

$g(f(x)) = g\left(\frac{2x+1}{3x-2}\right) = \frac{\frac{2x+1}{3x-2}-1}{\frac{2x+1}{3x-2}-2} = \frac{\frac{2x+1-3x+2}{3x-2}}{\frac{2x+1-6x+4}{3x-2}} = \frac{3-x}{5-4x}$

$D_{g\circ f}$ $\quad f \Rightarrow x \neq \frac{2}{3}$ $\quad g \Rightarrow x \neq 2 \Rightarrow \frac{2x+1}{3x-2} \neq 2$, $\quad 2x+1 \neq 6x-4$,

$4x \neq 5$, $\quad x \neq \frac{5}{4}$ $\quad \therefore D_{g\circ f} \Rightarrow x \in \mathbb{R}$, $x \notin \{\frac{2}{3}, \frac{5}{4}\}$

1:1? $\quad f(f^{-1}) \stackrel{?}{=} x$

$f\left(\frac{2x+1}{3x-2}\right) = \frac{2\left(\frac{2x+1}{3x-2}\right)+1}{3\left(\frac{2x+1}{3x-2}\right)-2} = \frac{4x+2+3x-2}{6x+3-6x+4} = \frac{7x}{7} = x$ $\quad f$ is 1:1.

II

$f(x) = \frac{(x+2)^2}{(x^2+5x+6)} = \frac{x+2}{x+3}$ $\quad D_f$ $x \in \mathbb{R}$, $x \notin \{-2, -3\}$

$g(x) = \frac{(2x-1)^2}{x^2-3x+2} = \frac{(2x-1)^2}{(x-2)(x-1)}$ $\quad D_g$ $x \in \mathbb{R}$ $x \notin \{2, 1\}$

$D_{\frac{f}{g}}$ $x \in \mathbb{R}$, $x \notin \{-2, -3, 1, \frac{1}{2}, 2\}$ $g(x) = 0$ when $x = \frac{1}{2}$ $\therefore$

$D_{g\circ f}$ $x \in \mathbb{R}$, $x \notin \{-2, -3, -4\}$ $\quad \frac{x+2}{x+3} \neq 2$ $\quad\quad \frac{x+2}{x+3} \neq 1$

$x+2 \neq 2x+6$ $\quad\quad x+2 \neq x+3$

$-4 \neq x$ $\quad\quad 2 \neq 3$ ALWAYS true

Extra Credit.

ANY self inversing, 1:1 function will do this Job. examples I, and $f(x) = \frac{2x+1}{3x-2}$ (from part I b)

all Range -5

all Domain of inverse -5

arithmetic errors -1

Comments.

In addition to deletions already marked delete "I A." & double pts to 80 for Section I. EXAM STILL appear too long!!

2 94
91

88
87 2
86
85
11 84 3
83
81 2
1 74
1 58

$\frac{1247}{15} = 83.13$

ANALYSIS CHAPT 1

NAME
Period KEY

I (80 pts ~~each~~) Find the formula of: (10pts) $\frac{f(x)}{g(x)}$, (10pts) $f(g(x))$, (10pts) $g(f(x))$, (10pts) $F(x)$, (10pts) $g^{-1}(x)$ for each of the following. STATE the domain of each, with its formula. STATE the ~~domain~~ and Range of $f(x)$ and $g(x)$. (20pts) Prove either f is or is not a 1:1 function. (10pts) ~~Prove either g is or is not a 1:1 function~~.

Use "A" as a query Before assigning the chart test.

A) $F(x) = \frac{1}{x+1}$ $g(x) = \frac{x}{x+1}$

B) $F(x) = \frac{2x+1}{3x-2}$ $g(x) = \frac{x-1}{x-2}$

II (20pts) ONLY Determine the Domain of:

$\frac{f(x)}{g(x)}$, ~~$f(g(x))$~~, $g(f(x))$

$f(x) = \frac{(x+2)^2}{x^2+5x+6}$ $g(x) = \frac{(2x-1)^2}{x^2-3x+2}$

EXTRA Credit (5pts)

List ~~three~~ ~~4~~ different functions ~~which differ~~ which satisfy the following condition:

$f(f(\ldots\ldots f(x)\ldots\ldots)) = x$

Explain fully!

ANALYSIS Chapt 7

I Solved

a) $a(x) = \frac{x+1}{x^2+1}$ $D_a = \mathbb{R}$

b) $b(x) = \frac{\sqrt{x+3}}{x^2+x}$ $x \geq -3$ $x \neq 0, -1$ $\mathbb{R}$

c) $c(x) = \frac{x\sqrt{x}}{x^2+5-x+6}$ $x \geq 0$ $\mathbb{R}$

d) $d(x) = \frac{x\sqrt{-x}}{x^2+5+x+6}$ $x \leq 0$ $x \neq -3, -2$

II Solved.

A) $f(x) = \frac{1}{x}$ $x \neq 0$ $\mathbb{R}$

$g(x) = x^2$ $\mathbb{R}$.

$f+g = \frac{1}{x} + x^2 = \frac{1+x^3}{x}$ $x \neq 0$ $\mathbb{R}$

$gf = x$ $\mathbb{R}$ $x \neq 0$

$\frac{f}{g} = \frac{\frac{1}{x}}{x^2} = \frac{1}{x^3}$ $x \neq 0$ $\mathbb{R}$

B) $f(x) = x-1$ $D_f \Rightarrow \mathbb{R}$

$g(x) = x^2-1$ $D_g \Rightarrow \mathbb{R}$.

$f+g = x^2+x-2$ $D \Rightarrow \mathbb{R}$

$gf = x^3-x^2-x+1$ $D \Rightarrow \mathbb{R}$.

$\frac{f}{g} = \frac{x-1}{x^2-1} = \frac{1}{x+1}$ $D \Rightarrow \mathbb{R}$ $x \neq \pm 1$

III solved.

$f(x) = \frac{(x+3)^2}{x(3x^2-5x-2)} = \frac{(x+3)^2}{x(3x+1)(x-2)}$ $\mathbb{R}$ $D_f = x \neq 0, -\frac{1}{3}, 2$

(intersection of Domains or union of Restrictions)

$g(x) = \frac{(x+3)^2}{12x^2+4x+2} = \frac{(x+3)^2}{(4x+1)(3x+2)}$ $\mathbb{R}$, $D_g = x \neq -\frac{1}{4}, -\frac{2}{3}$

$g = 0$ $x = -3$.

$$\begin{cases} D_{fg} \Rightarrow \mathbb{R} \; x \neq 0, \frac{-1}{3}, 2, -\frac{1}{4}, -\frac{2}{3} \\ D_{\frac{f}{g}} \Rightarrow x = 0, \frac{-1}{3}, 2, -\frac{1}{4}, -\frac{2}{3}, -3. \end{cases}$$

IV solved.

A) $f(x) = x-1$ $\mathbb{R}$

$g(x) = \sqrt{x}$ $x \geq 0$

$x-1 \geq 0$

$x \geq 1$

$D_{g \circ f} = \mathbb{R}$, $x \geq 1$

$g \circ f = \sqrt{x-1}$

b) $f(x) = -x^2$ $\mathbb{R}$

$g(x) = \sqrt{x}$ $x > 0$

$-x^2 \geq 0$

$x^2 \leq 0$

$x < 0$

$D_{g \circ f}$ $\{0\}$

$g \circ f = \sqrt{-x^2}$

c) $f(x) = \frac{1}{x-1}$ $\mathbb{R}$ $x \neq 1$

$g(x) = \frac{x+1}{x}$ $\mathbb{R}$ $x \neq 0$

$D_{g \circ f} = \mathbb{R}$ $x \neq 1$

$g \circ f = \frac{\frac{1}{x-1} + 1}{\frac{1}{x-1}} = \frac{\frac{1+x-1}{x-1}}{\frac{1}{x-1}} =$

$= x$

V $f(x) = \frac{2x-1}{3x+4}$ $\quad y = \frac{2x-1}{3x+4}$ $\quad x = \frac{2y-1}{3y+4}$ $\quad 3xy+4x = 2y-1$

$3xy - 2y = -4x - 1$

$(3x-2)y =$

$y = \frac{-4x-1}{3x-2} = \frac{4x+1}{2-3x} = f^{-1}$

$f(f^{-1}) = \frac{2\left(\frac{4x+1}{2-3x}\right)-1}{3\left(\frac{4x+1}{2-3x}\right)+4} = \frac{\frac{8x+2+3x-2}{2-3x}}{\frac{12x+3+8-12x}{2-3x}} = \frac{11x}{11} = x$

$f(x)$ is 1:1

VI $f(x) = \frac{x+2}{3x-1}$ $\quad y = \frac{x+2}{3x-1}$ $\quad x = \frac{y+2}{3y-1}$ $\quad 3xy - x = y + 2$

$3xy - y = x + 2$

$y(3x-1) =$

$y = \frac{x+2}{3x-1}$ f is Self inversion.

Extra. Sets Total.

a) $f(x) = \frac{x}{x}$ $\quad x \neq 0$

Not 1:1 (constant fctn)

$x \neq 0$ because 0 in denominator undefined.

b) $g(x) = \frac{\sqrt{x}}{x}$ $\quad x > 0$ $\quad y = \frac{1}{\sqrt{x}}$

$g(x) = \frac{1}{\sqrt{x}}$ $\quad x > 0$ $\quad x = \frac{1}{\sqrt{y}}$

$\sqrt{y} = \frac{1}{x}$

$g(g^{-1}x) = \frac{\sqrt{\frac{1}{x^2}}}{\frac{1}{x^2}} = \frac{\frac{1}{x}}{\frac{1}{x^2}} = x$ $\quad y = \frac{1}{x^2} = f(x)$

g is 1:1 $\quad x > 0$ because square root not defined when negative. and division by zero undefined.

c) $h(x) = \frac{x+B}{x}$ $\quad B \neq 0$ $\quad x \neq 0$

$x = \frac{y+B}{y}$ $\quad h$ is 1:1

$xy - y = B$ $\quad B \neq 0$ so $h(x)$ is not a constant.

$y(x-1) = B$ $\quad x \neq 0$ since division by zero is undefined.

$h^{-1}(x) = y = \frac{B}{x-1}$ (Note $x \neq 1$ in h^{-1}. since division by zero undefined

$\frac{\frac{B}{x-1} + B}{\frac{B}{x-1}} = \frac{\frac{B + Bx - B}{x-1}}{\frac{B}{x-1}} = \frac{Bx}{B} = x$

ANALYSIS CHAPT 1

NAME Key.

Period

I Determine the Domains over the Real numbers for each of the following fcns. (5pts ea)

a) $a(x) = \frac{x+1}{x^2+1}$ b) $b(x) = \frac{\sqrt{x+3}}{x^2+x}$ c) $c(x) = \frac{x\sqrt{x}}{x^2+5x+6}$

d) $d(x) = \frac{x\sqrt{-x}}{x^2+5x+6}$

II Given $f(x)$ and $g(x)$ find $f+g$, gF and $\frac{F}{g}$. STATE The Domains for $f(x)$, $g(x)$, $f+g$, gF and $\frac{F}{g}$. (10pts ea)

a) $f(x) = \frac{1}{x}$ $g(x) = x^2$ b) $f(x) = x-1$ $g(x) = x^2-1$

III ONLY STATE Domain of fg and $\frac{F}{g}$ (10pts total)

$f(x) = \frac{(x+3)^2}{x(3x^2-5x-2)}$ $g(x) = \frac{(x+3)^2}{12x^2+11x+2}$

IV Find the Domain of $g \circ f$ and give the formula for $g \circ f$. (10pts ea)

a) $f(x) = x-1$, $g(x) = \sqrt{x}$ b) $f(x) = -x^2$, $g(x) = \sqrt{x}$ c) $f(x) = \frac{1}{x-1}$, $g(x) = \frac{x+1}{x}$

V Show either $f(x)$ is or is NOT a 1:1 function (10pts)

$f(x) = \frac{2x-1}{3x+4}$

VI SHOW either $f(x)$ is or is NOT self INVERSIVE (10pts)

$f(x) = \frac{x+2}{3x-1}$

EXTRA (5pts) Which of the following ARE 1:1 fcns? Explain the Restrictions on each function.

a) $f(x) = \frac{x}{x}$ $x \neq 0$ b) $g(x) = \frac{\sqrt{x}}{x}$ $x > 0$

c) $h(x) = \frac{x+B}{x}$ $B \neq 0$ $x \neq 0$

I

$f(x) = \frac{2x+3}{5x-2}$ $g(x) = \frac{2x+3}{x}$

15

A) D_f $x \in \mathbb{R}$ $x \neq \frac{2}{5}$ D_g $x \in \mathbb{R}$ $x \neq 0$

R_f $f(x) \in \mathbb{R}$ $f(x) \neq \frac{2}{5}$ R_g $g(x) \in \mathbb{R}$ $g(x) \neq 2$

B) $x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2} = f^{-1} = l(x)$ 10

$D_{f^{-1}}$ $x \in \mathbb{R}$ $x \neq \frac{2}{5}$

$x = \frac{2z+3}{z}$

$xz - 2z = 3$

$(x-2)z = 3$

$z = \frac{3}{x-2} = g^{-1}(x) = m(x)$ 10

$D_{g^{-1}}$ $x \in \mathbb{R}$ $x \neq 2$

$g(x) \overset{\text{set}}{=} 0$

$\frac{2x+3}{x} \overset{\text{set}}{=} 0$

$x = -\frac{3}{2}$ -5

-5

$h(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{5x-2} \cdot \frac{x}{2x+3} = \frac{x}{5x-2}$ 10 D_h $x \in \mathbb{R}$, $x \neq \{-\frac{3}{2}, 0, \frac{2}{5}\}$

$J(x) = f(g(x)) = f\left(\frac{2x+3}{x}\right) = \frac{2\left(\frac{2x+3}{x}\right)+3}{5\left(\frac{2x+3}{x}\right)-2} = \frac{4x+6+3x}{10x+15-2x} = \frac{7x+6}{8x+15}$

D_J $x \in \mathbb{R}$ $x \neq \{0, -\frac{15}{8}\}$ 10

$\frac{2x+3}{x} \overset{\text{set}}{=} \frac{2}{5}$

$10x + 15 = 2x$

$8x = -15$

$x = -\frac{15}{8} \in S$

$K(x) = g(f(x)) = g\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{\frac{2x+3}{5x-2}} =$

$\frac{4x+6+15x-6}{2x+3} = \frac{19x}{2x+3} = K(x)$

$f(x) \overset{\text{set}}{=} 0 \overset{\text{SJ}}{=} \frac{2x+3}{5x-2} \Rightarrow x = -\frac{3}{2}$

D_K $x \in \mathbb{R}$ $x \neq \{-\frac{3}{2}, \frac{2}{5}\}$ 10

C $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ Since $f = f^{-1}$ SPTS

we need only show one portion $\therefore$ SPTS

$f\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2} = \frac{4x+6+15x-6}{10x+15-10x+4} = \frac{19x}{19} = x$

$\therefore$ f is 1:1 f SPTS

II $f(x) = \dfrac{(x+2)^2}{x^2+5x+6}$ $\quad$ $\dfrac{(x+2)^2}{(x+3)(x+2)}$ $\quad$ $g(x) = \dfrac{(2x-1)^2}{x^2-3x+2}$ $\quad$ $\dfrac{(2x-1)^2}{(x-2)(x-1)}$

D_f $x \in \mathbb{R}$ $x \notin \{-3, -2\}$ $\quad$ D_g $x \in \mathbb{R}$ $x \notin \{1, 2\}$

$D_{\frac{f}{g}}$ $\quad g(x) \overset{\text{set}}{=} 0 \overset{\text{set}}{=} \dfrac{(2x-1)^2}{x^2-3x+2} \Rightarrow x = \frac{1}{2}$

$D_{\frac{f}{g}}$ $x \in \mathbb{R}$ $x \notin \{-3, -2, \frac{1}{2}, 1, 2\}$

$D_{g \circ f}$ $\quad f(x) \overset{\text{set}}{=} -1 \overset{\text{set}}{=} \dfrac{(x+2)^2}{(x+3)(x+2)} \Rightarrow 1 = \dfrac{x+2}{x+3}$

$x+3 = x+2$

$3 = 2$ Never true

$\therefore$ no additional restrictions

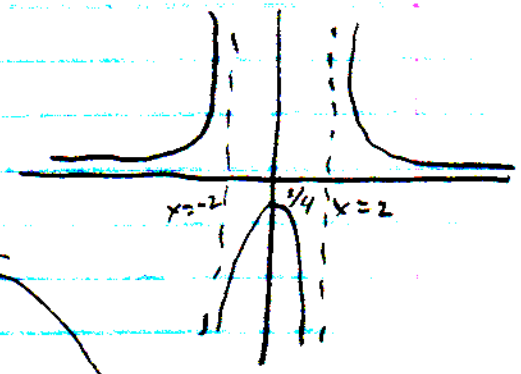
$\dfrac{x+2}{x+3} \overset{\text{set}}{=} 2$

$x+2 = 2x+6$

$x = -4$

$\therefore D_{g \circ f}$ $x \in \mathbb{R}$ $x \notin \{-4, -3, -2\}$

Extra credit $\quad f(x) = \dfrac{1}{x^2-4}$



D_f $x \in \mathbb{R}$ $x \notin \{\pm 2\}$

R_f $f(x) > 0$ or $f(x) < -\frac{1}{4}$

$x = \dfrac{1}{z^2-4}$ $\quad xz^2 - 4x = 1$

$xz^2 = 1 + 4x$

$z = \pm\sqrt{\dfrac{1+4x}{x}}$

$\dfrac{1+4x}{x} > 0$

Case I: $1+4x \geq 0$ & $x > 0$ or $1+4x \leq 0$ & $x < 0$

$x \geq -\frac{1}{4}$ $\quad x > 0$ $\quad$ or $\quad x \leq -\frac{1}{4}$ $\quad x < 0$

$x > 0$ $\qquad$ $x \leq -\frac{1}{4}$

Except $x \neq 0$ $\quad$ $x > 0$

Since this is effect on Domain of f^{-1} then it applies to Range of $f(x)$

78

NAME KEY

Period DATE

ANALYSIS Chapt 1

I (80 pts total) Given $f(x)=\frac{2x+3}{5x-2}$ $g(x)=\frac{2x+3}{x}$

A) STATE Domain + Range of $f(x)$, $g(x)$. (15 pts)

B) Find the formula for And state the Domain of $h(x)$, $J(x)$, $K(x)$, $\ell(x)$, $m(x)$ if: (10 pts ea)

$h(x)=\frac{f(x)}{g(x)}$ $J(x)=f(g(x))$ $K(x)=g(f(x))$

$\ell(x)=f^{-1}(x)$ $m(x)=g^{-1}(x)$

C) What is required to prove a polynomial fctn is 1:1? Prove $f(x)$ is or is not a 1:1 fctn. (15 pts total)

I

Given $f(x)=\frac{(x+2)^2}{x^2+5x+6}$ $g(x)=\frac{(2x-1)^3}{x^2-3x+2}$

STATE only the Domain of: (10 pts ea)

A) $\frac{f(x)}{g(x)}$ B) $g(f(x))$

Extra credit (5 pts.)

STATE Range and Domain and sketch the fctn:

$f(x)=\sqrt{\frac{1}{x^2-4}}$

2/10/78

87

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105·00 +
105·00 +
98·00 +
96·00 +
86·00 +
84·00 +
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1978 Good exam.
Giving two days prior to exam cleared up much for the students. Test length is about right (possibly a little shorter but no need to lengthen)

I If a fctn associates one item from a set called the Domain with a unique element from a set called the Range.

II (3 pts ea)

A) fctn D $\{0,1,2,3\}$ R $\{1,2\}$

B) Relation

C) fctn D $x \in \mathbb{R}$ R $y \geq 1$

D) Relation

E) fctn D $x \in \mathbb{R}$ R $y = 4$

III

$$f(x) = \frac{2x+3}{5x-2} \qquad g(x) = \frac{2x+3}{x}$$

A) (10)

D_f $x \in \mathbb{R}$ $x \neq \frac{2}{5}$

R_f $f(x) \in \mathbb{R}$ $f(x) \neq \frac{2}{5}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2}$

D_g $x \in \mathbb{R}$ $x \neq 0$ (10)

R_g $g(x) \in \mathbb{R}$ $g(x) \neq 2$

$x = \frac{2z+3}{z}$

$xz - 2z = 3$

$z = \frac{3}{x-2}$

$\frac{2x+3}{x} \overset{\text{set}}{=} \frac{2}{5}$

$10x + 15 = 2x$

$8x = -15$

$x = -\frac{15}{8}$

B)

① $\frac{f(x)}{g(x)} = \frac{\frac{2x+3}{5x-2}}{\frac{2x+3}{x}} = \frac{x}{5x-2}$ (C0)

$x \in \mathbb{R}$ $x \neq \{\frac{2}{5}, 0, -\frac{3}{2}\}$

$\frac{2x+3}{x} \overset{\text{set}}{=} 0 \Rightarrow x = -\frac{3}{2}$

② $f\left(\frac{2x+3}{x}\right) = \frac{2\left(\frac{2x+3}{x}\right)+3}{5\left(\frac{2x+3}{x}\right)-2} = \frac{4x+6+3x}{10x+15-2x} = \frac{7x+6}{8x+15}$ (10 pts)

D_2 $x \in \mathbb{R}$ $x \neq \{0, -\frac{15}{8}\}$

C) $D_{g(f(x))}$ $\frac{2x+3}{5x-2} \overset{\text{set}}{=} 0 \Rightarrow x = -\frac{3}{2}$ $x \in \mathbb{R}$ $x \neq \{\frac{2}{5}, -\frac{3}{2}\}$ 6 pts

IV A) (5) If a fctn is 1:1 its inverse is also a fctn, or if a fctn is 1:1 each element in Range is associated with a unique element from the Domain.

(5) B) $f(f^{-1}(x)) = x = f^{-1}(f(x))$

C) Since $f(x)$ is self inversing only one side is needed

$$f\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2} = \frac{\frac{4x+6+15x-6}{5x-2}}{\frac{10x+15-10x+4}{5x-2}} = \frac{19x}{19} = x$$

f is 1:1. (5)

II $f(x) = \dfrac{(x+2)^2}{(x+2)(x+3)} = \dfrac{x+2}{x+3}$ $g(x) = \dfrac{(2x-1)^2}{x^2-3x+2}$ $(x-1)(x-2)$

A) $\dfrac{(2x-1)^2}{x^2-3x+2} \overset{\text{Set}}{=} 0$ $D_{\frac{f}{g}}$ $x \in \mathbb{R}\ x \neq \{-2, -3, \frac{1}{2}, 1, 2\}$ 5

$x = \frac{1}{2}$

B) $\dfrac{x+2}{x+3} \overset{\text{Set}}{=} 1$ $\dfrac{x+2}{x+3} = 2$ $D_{g(f(x))}$ $x \in \mathbb{R}\ x \neq \{-4, -3, -2\}$ 5

$x+2 = x+3$ $x+2 = 2x+6$

impossible $x = -4$

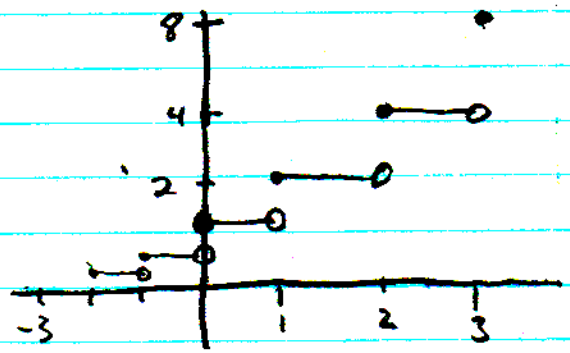
C) $\sqrt{\frac{1}{f(x)}} =$ $(x+2)(x+3) > 0$

$x+2 > 0$ & $x+3 > 0$ or $x+2 < 0$ & $x+3 < 0$

5 D $x \in \mathbb{R}$; $x < -3$ or $x > -2$ $x > -2$ $x > -3$ $x < -2$ $x < -3$

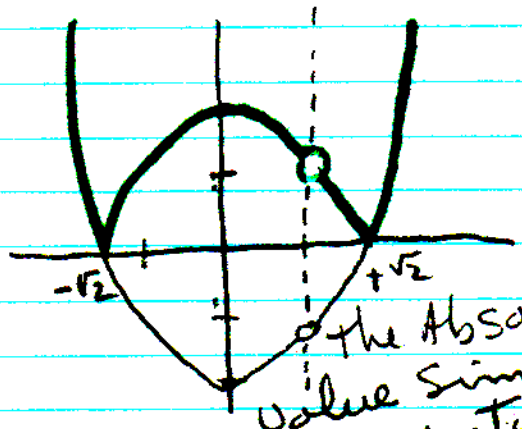
$x > -2$ or $x < -3$

VI $y = 2^{[\![x]\!]}$ $-2 \le x \le 3$ 5



x	$[\![x]\!]$	y
$-2 \le x < -1$	-2	$\frac{1}{4}$
$-1 \le x < 0$	-1	$\frac{1}{2}$
$0 \le x < 1$	0	1
$1 \le x < 2$	1	2
$2 \le x < 3$	2	4
$x = 3$	3	8

Extra $y = \left|\dfrac{(x^2-2)(x-1)}{(x-1)}\right| = x^2 - 2$ $x \ge \sqrt{2}$



the Absolute value simply flips the neg portion to top.

1978

2495 = 80.5

$$\left|\frac{(x^2-2)(x-1)}{(x-1)}\right| = \lambda$$

Honors ANALYSIS CHAPT 1 NAME Key 1979

Per ___ DATE ___

I Define: function. (Do not use examples) (5pts)

II Which of the following RELATIONS are functions? For EACH which is A fctn, STATE Domain & Range. (3pts ea)

A) $\{(0,1), (1,2), (2,1), (3,2)\}$

B) $\{(1,0), (2,1), (1,2), (2,3)\}$

C) $y = x^2 + 1$

D) $x^2 + y^2 = 4$

E) $y = \sqrt{16}$

III given $f(x) = \frac{2x+3}{5x-2}$ & $g(x) = \frac{2x+3}{x}$

A) STATE the Domain & Range for $f(x)$, $g(x)$ (2opts)

B) DeTeRmine the formula for & the Domain of

① $h(x) = \frac{f(x)}{g(x)}$ ② $L(x) = f(g(x))$ (10pts ea)

C) DeTeRmine the Domain of $g(f(x))$. (5pts)

IV A) What does it mean to say a fctn is 1:1? (5pts)

B) What is Required to PRove a fctn is 1:1? (5pts)

C) If $f(x) = \frac{2x+3}{5x-2}$, PRove $F(x)$ is or is not 1:1. (5pts)

V Given $f(x) = \frac{(x+2)^2}{x^2+5x+6}$ and $g(x) = \frac{(2x-1)^2}{x^2-3x+2}$ DeTeRmine the Domain of: (5pts ea)

A) $\frac{f(x)}{g(x)}$ B) $g(f(x))$ C) $\sqrt{\frac{1}{f(x)}}$

VI Sketch: $y = 2^{[\![x]\!]}$ for $-2 \le x \le 3$ (5pts)

EXTRA Credit

Sketch: $y = \left| \frac{(x^2-2)(x-1)}{(x-1)} \right|$

I A) $A(x) = \sqrt{9-x^2}$

$D_A \quad x \in \mathbb{R} \quad |x| \le 3$

$R_A \quad 0 \le A(x) \le 3$

B) $B(x) = \sqrt{\frac{x}{x-1}}$ $\quad x = \sqrt{\frac{z}{z-1}}$

$D_B \quad x \in \mathbb{R} \quad x > 1$ or $x \le 0$ $\quad x^2 z - x^2 = z$

$(x^2-1)z = x^2$

$z = \frac{x^2}{x^2-1}$

$R_B \quad B(x) \ge 0, \ B(x) \ne 1$

C) $C(x) = \sqrt{25}$

$D_C \quad x \in \mathbb{R}$

$R_C \quad C(x) = 5$

II $f(x) = x^2 - 1$

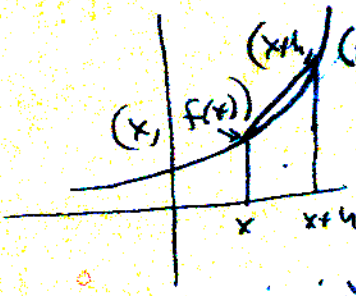
a) $f(x+h) - f(x) = (x+h)^2 - 1 - x^2 + 1$

$= x^2 + 2xh + h^2 - x^2$

$= 2xh + h^2$

b) $\frac{f(x+h)-f(x)}{h} = 2x + h$

c)



$\frac{y_2 - y_1}{x_2 - x_1} = m$

This expression is the slope of the chord as shown.

III $f(x) = \frac{ax-b}{a-cx} \Rightarrow x = \frac{az-b}{a-cz}$

$f(x) = \frac{ax-b}{a-cx} \overset{\text{let}}{=} k$

$ax - cxz = az - b$

$ax - b = ak - ckx$

$-(cx+a)z = -(ax+b)$

$x(a+ck) = ak+b$

$z = \frac{ax+b}{cx+a} = f^{-1}(x)$

$x = \frac{ak+b}{a+ck}$

$a \ne cx \Rightarrow x \ne \frac{a}{c}$

$cx + a \ne 0 \Rightarrow x \ne \frac{-a}{c}$

if $bc = a^2$ $\quad b = \frac{a^2}{c}$ $\quad f(x) = \frac{ax - \frac{a^2}{c}}{a-cx} = \frac{\frac{acx - a^2}{c}}{a-cx} = \frac{-a(a-cx)}{c(a-cx)} = -\frac{a}{c} = \text{constant}$

ie no univ. fctn $\therefore$ $a^2 \ne bc$ is also required.

IV $f(x) = \frac{2x+3}{5x-2}$ $\quad g(x) = \frac{2x+3}{x}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x - 2z = 3$

$(5x-2)z = 3 + 2x$

$f^{-1}(x) \quad z = \frac{3+2x}{5-2x}$

$x = \frac{2z+3}{z}$

$(x-2)z = 3$

$z = \frac{3}{x-2}$

a)

$D_f \quad x \in \mathbb{R} \quad x \ne \frac{2}{5}$

$R_f \quad f(x) \in \mathbb{R} \quad f(x) \ne \frac{2}{5}$

$D_g \quad x \in \mathbb{R} \quad x \ne 0$

$R_g \quad g(x) \in \mathbb{R} \quad g(x) \ne 2$

B ① $h(x) = \frac{f(x)}{g(x)}$ $\quad D_h \quad x \in \mathbb{R} \quad x \notin \left\{0, \frac{2}{5}, -\frac{3}{2}\right\}$

$g(x) \overset{\text{let}}{=} 0 \Rightarrow x = -\frac{3}{2}$

② $J(x) = g(f(x))$ $\quad D_J \quad x \in \mathbb{R} \quad x \notin \left\{\frac{2}{5}, -\frac{3}{2}\right\}$

$f(x) = \frac{2x+3}{5x-2} \overset{\text{let}}{=} 0 \Rightarrow x = -\frac{3}{2}$

③ $K(x) = f(g(x))$ $\quad D_K \quad x \in \mathbb{R} \quad x \notin \left\{0, \frac{2}{5}, -\frac{15}{8}\right\}$

$g(x) = \frac{2x+3}{x} \overset{\text{let}}{=} \frac{2}{5}$

$10x + 15 = 2x$

$8x = -15$

$x = -\frac{15}{8}$

IV

c) $f(x) = \frac{2x+3}{5x-2} = f^{-1}(x)$ $f(f^{-1}(x)) = f(f(x)) = x$ if f is 1:1

since $f = f^{-1}$ we need only to show one side.

$$f\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2} = \frac{4x+6+15x-6}{10x+15-10x+4} = \frac{19x}{19} = x$$

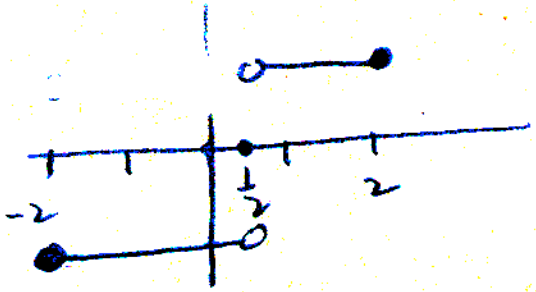
$\therefore$ f is 1:1

V $f(x) = \text{Sgn}(2x-1)$ $-2 \le x \le 2$

$2x-1 \overset{\text{Let}}{=} 0$ when $x < \frac{1}{2}$ $2x-1 < 0$ $\therefore$ $\text{Sgn}(2x-1) = -1$ $x < \frac{1}{2}$

$2x = 1$ $x = \frac{1}{2}$ $2x-1 = 0$ $\therefore$ $\text{Sgn}(2x-1) = 0$

$x = \frac{1}{2}$ $x > \frac{1}{2}$ $2x-1 > 0$ $\therefore$ $\text{Sgn}(2x-1) = 1$



Extra as $h \to 0$ we are approaching the pt $(x, f(x))$ actually when h is very close to zero we have the tangent to the curve at $(x, f(x))$.

1980

This was an excellent exam. Student took sloppy notes which accounts for about 7 pts of mistakes!

$\frac{1652}{21} = 78.7$

Honors Analysis Chapt 1 Name Key

Per ___ Date 1980

I State the Domain & Range for the following: (5pts ea)

a) $A(x) = \sqrt{9-x^2}$ b) $B(x) = \sqrt{\frac{x}{x-1}}$ c) $C(x) = \sqrt{25}$

II Given $f(x) = x^2 - 1$ (5pts)

(5pts) a) find $f(x+h) - f(x)$ b) find $\frac{f(x+h)-f(x)}{h}$

c) Show and Explain the geometric meaning of $\frac{f(x+h)-f(x)}{h}$. (10pts)

III Find the inverse fctn of $f(x) = \frac{ax-b}{a-cx}$. (Be very clear about the restrictions on a, b, and c. Show how you arrive at your conclusions) 15 pts

IV Given $f(x) = \frac{2x+3}{5x-2}$, $g(x) = \frac{2x+3}{x}$

A) State the Domain AND Range for f(x), g(x) (20pts total)

B) Determine Domain ONLY for: (total 15pts)

① $h(x) = \frac{f(x)}{g(x)}$ ② $J(x) = g(f(x))$ ③ $K(x) = f(g(x))$

C) Prove f(x) is or is not a 1:1 fctn. Explain. (5pts)

V Graph $f(x) = \text{Sgn}(2x-1)$ for $-2 \le x \le 2$ (10pts)

Extra Credit (5pts)

Consider Section II Again. If the value, h, is allowed to approach (But Never Actually Equal) Zero, what would be the geometric meaning of the expression: $\frac{f(x+h)-f(x)}{h}$?

I a) fctn

D: $\{0, 1, 2, 3\}$

R: $\{1, 2\}$

b) Relation

D: $\{1, 2\}$

R: $\{0, 1, 2, 3\}$

c) $Y = x^2 + 1$ fctn

D: $x \in \mathbb{R}$

D: $Y \in \mathbb{R}\ Y \geq 1$

D) $x^2 + y^2 = 4$ relation

D $|x| \leq 2$

R $|y| \leq 2$

E) $Y = \sqrt{16}$

D $x \in \mathbb{R}$

R $y = 4$

f) $Y = \sqrt{9 - x^2}$ fctn

D: $|x| \leq 3$

R: $0 \leq Y \leq 3$

g) $Y = \sqrt{\frac{x}{x^2 - 1}}$ fctn

D $x \in \{x > -1, x \neq 1\}$

$-1 < x \leq 0,\ x > 1$

II $f(x) = \frac{2x+3}{5x-2}$ $g(x) = \frac{2x+3}{x}$

A) D_f $x \in \mathbb{R}\ x \neq \frac{2}{5}$

R_f $f(x) \in \mathbb{R}\ f(x) \neq \frac{2}{5}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x - 2)z = 2x + 3$

$z = \frac{2x+3}{5x-2}$

D_g $x \in \mathbb{R}\ x \neq 0$

R_g $g(x) \in \mathbb{R}\ g(x) \neq 2$

$x = \frac{2z+3}{z}$

$xz - 2z = 3$

$z = \frac{3}{x-2}$

B) $\frac{f}{g} = \frac{\frac{2x+3}{5x-2}}{\frac{2x+3}{x}} = \frac{x}{5x-2} = h(x)$

$D_{\frac{f}{g}}$ $x \in \mathbb{R}\ x \neq \{\frac{2}{5}, 0, -\frac{3}{2}\}$

$\frac{2x+3}{x} \overset{\text{set}}{=} 0 \Rightarrow x = -\frac{3}{2}$

$L(x) = f(g(x)) = \frac{2\left(\frac{2x+3}{x}\right) + 3}{5\left(\frac{2x+3}{x}\right) - 2} =$

$\frac{4x + 6 + 3x}{10x + 15 - 2x} = \frac{7x+6}{8x+15}$

D_L $x \in \mathbb{R}\ x \neq \{0, -\frac{15}{8}\}$

$\frac{2x+3}{x} \overset{\text{set}}{=} \frac{2}{5}$

$10x + 15 = 2x$

$8x = -15$

$x = -\frac{15}{8}$

C) $g(f(x))$

$D_{g(f(x))}$ $x \in \mathbb{R}\ x \neq \{\frac{2}{5}, -\frac{3}{2}\}$

$\frac{2x+3}{5x-2} \overset{\text{set}}{=} 0 \Rightarrow x = -\frac{3}{2}$

III $f(x) = \dfrac{(x+2)^2}{(x+5x+6)} = \dfrac{x+2}{x+3}$ $g(x) = \dfrac{(2x-1)^2}{(x-1)(x-2)}$

30 PTS

A) $D_{\frac{f}{g}}$ $x \in \mathbb{R}$ $x \neq \{-2, -3, 1, 2, \frac{1}{2}\}$

30 PTS

B) $D_{g(f(x))}$ $x \in \mathbb{R}$ $x \neq \{-2, -3, -4\}$

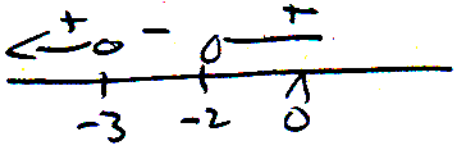
$\dfrac{x+2}{x+3} \neq 1$ $\dfrac{x+2}{x+3} = 2$

$x+2 = x+3$ $x+2 = 2x+6$

not $x = -4$.

C) $D_{\sqrt{\frac{1}{f(x)}}}$ $x \in \mathbb{R}$ $x < -3$, or $x \geq -2$

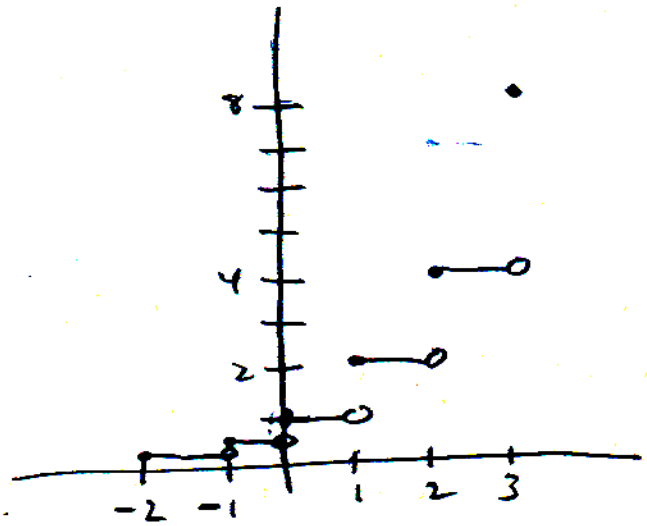
$\dfrac{+}{(x+2)(x+3)}$ $\therefore$ $(x+2)(x+3) > 0$



IV

$y = 2^{[\![x]\!]}$ $-2 \leq x \leq 3$

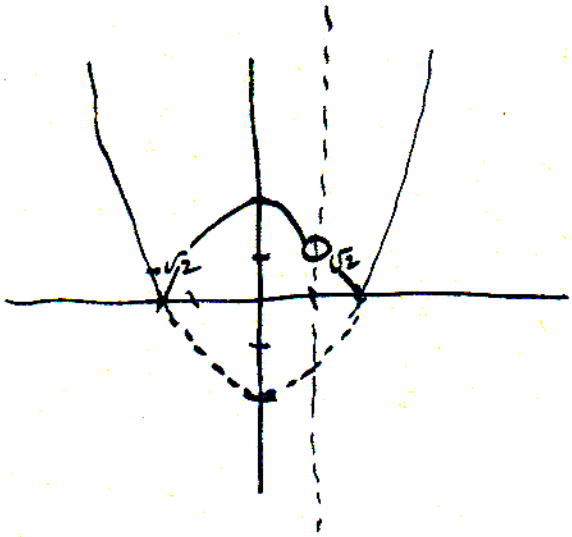
$[\![x]\!]$	y
$-2 \leq x < -1$	-2	$\frac{1}{4}$
$-1 \leq x < 0$	-1	$\frac{1}{2}$
$0 \leq x < 1$	0	1
$1 \leq x < 2$	1	2
$2 \leq x < 3$	2	4
3	3	8

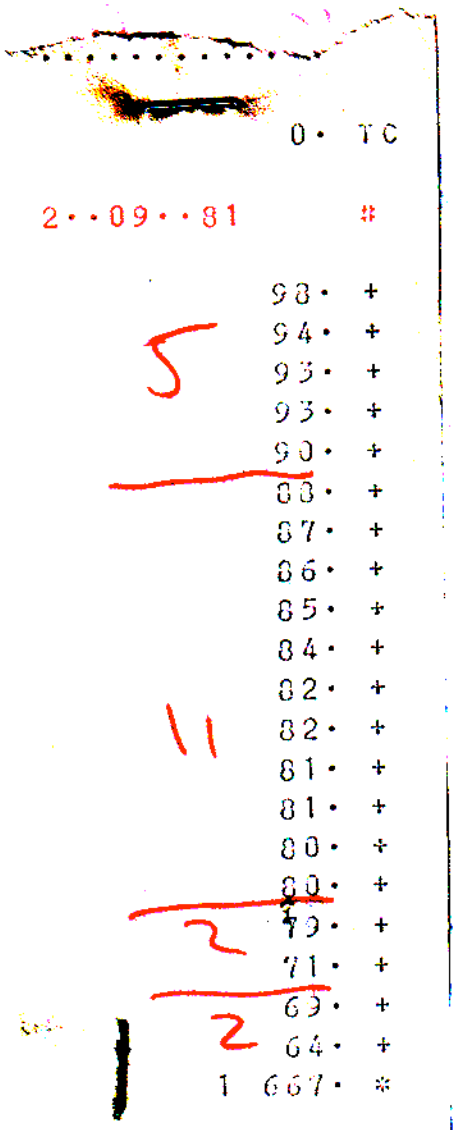


Extra $y = \left|\dfrac{(x^2-2)(x-1)}{(x-1)}\right| = |x^2-2|$

Just flip neg portion of curve up.

Notice $x = 1$ not Defined.





5

11

2

2

good exam
make changes
shown on
other side

No other
changes
necessary

HONORS ANALYSIS CHAPT 1 EXAM NAME Key 1981

READ CAREFULLY AND FOLLOW DIRECTIONS!

I ~~Which of the following are fctns?~~ For each of the following, State which are fctns and which are not fctns. STATE Domain AND Range. (5 pts ea)

a) $\{(0,1),(1,2),(2,1),(3,2)\}$ b) $\{(1,0),(2,1),(1,2),(2,3)\}$

c) $y = x^2 + 1$ d) $x^2 + y^2 = 4$ e) $y = \sqrt{16}$

f) $y = \sqrt{9 - x^2}$ g) $y = \sqrt{\sum_{x=1}^{x} 1}$ ⇐ FIND ONLY Domain on g.

II Given $f(x) = \frac{2x+3}{5x-2}$; $g(x) = \frac{2x+3}{x}$

A) STATE specifically: D_f, R_f, D_g, R_g (20 pts)

B) DETERMINE the formula for AND ONLY the Domain (NOT the Range!) of $h(x)$ and $L(x)$ (total 20 pts)

Given: $h(x) = \frac{f(x)}{g(x)}$ $L(x) = f(g(x))$

C) Determine ONLY the Domain of $g(f(x))$ (5 pts)

III Given $f(x) = \frac{(x+2)^2}{(x^2+5x+6)}$; $g(x) = \frac{(2x-1)^2}{x^2-3x+2}$ Determine ONLY the Domain of:

a) $\frac{f(x)}{g(x)}$ b) $g(f(x))$ c) $\sqrt{\frac{1}{f(x)}}$ (total 15 pts)

IV Sketch $y = 2^{[\![x]\!]}$ for $-2 \le x \le 3$ (5 pts)

EXTRA 5 pts — MAKE A very clear sketch of $y = \left| \frac{(x^2-2)(x-1)}{(x-1)} \right|$

I 5 pts ea

A) $A(x) = \dfrac{x\sqrt{x}}{(x-2)(x-3)}$

only $x \in \mathbb{R}$, $x \neq \{-3\}$ — D_A $x \in \mathbb{R}$ $x \geq 0$

-3

B) $B(x) = \dfrac{\sqrt{x+3}}{x(x+1)}$

D_B $x \in \mathbb{R}$ $x \geq -3$, $x \neq \{0, -1\}$

C) $C(x) = \sqrt{x^2+8x+12}$

$= \sqrt{(x+2)(x+6)}$

+ 0 − 0 +

-6 -2 0

D_C $x \in \mathbb{R}$ $x \geq -2$ only -2

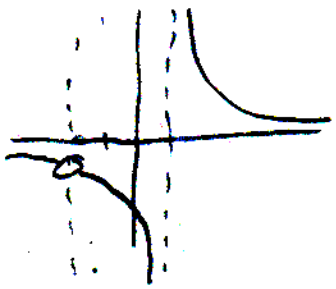
or

$x \leq -6$

II 10 pts ea

A) $A(x) = \dfrac{(x+2)}{(x-1)(x+2)}$

D_A $x \in \mathbb{R}$ $x \neq \{-1, -2\}$



$x \to \infty$ $A(x) \to 0^+$

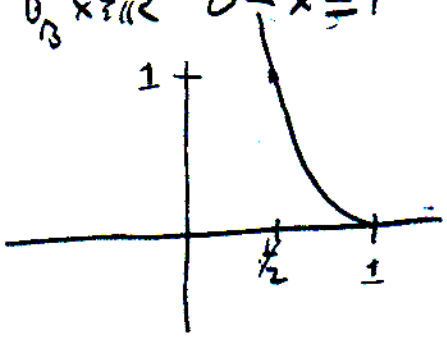
$x \to 1^+$ $A(x) \to +\infty$

$x \to 1^-$ $A(x) \to -\infty$

$x \to -\infty$ $A(x) \to 0^-$

B) $B(x) = \sqrt{\dfrac{1}{x} - 1}$

D_B $x \in \mathbb{R}$ $0 < x \leq 1$



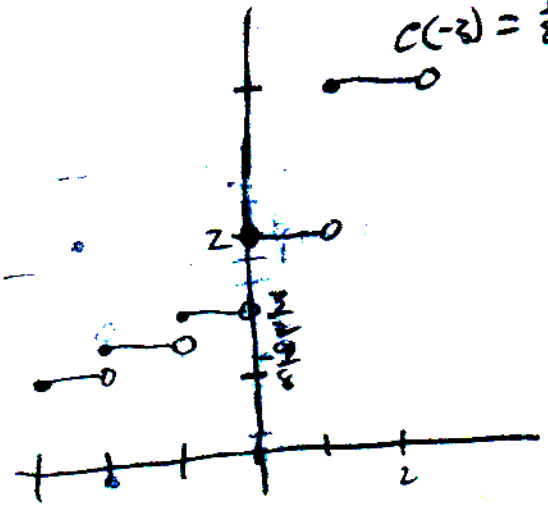
$x \to 0^+$ $B(x) \to +\infty$

$x = \frac{1}{2}$ $B(x) = 1$

$x = 1$ $B(x) = 0$

II C) $C(x) = 2^{|x+1|} + 1$ $-3 \leq x \leq 2$

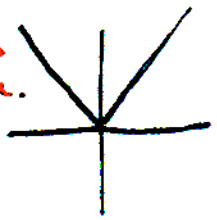
$C(-3) = \frac{1}{8} + 1$



$C(x) > 1 \; \forall x$

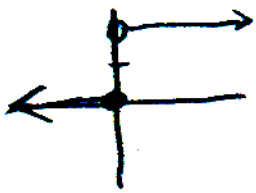
III A) $A(x) = x \operatorname{Sgn}(x)$ 10 pts ea.

$$A(x) = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$$



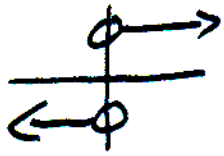
B) $B(x) = (\operatorname{Sgn}(x))^2 + \operatorname{Sgn} x$

$$B(x) = \begin{cases} 2 & x > 0 \\ 0 & x = 0 \\ 0 & x < 0 \end{cases}$$



C) $C(x) = \dfrac{x}{|x|}$

$$C(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$



IV $f(x) = \frac{2x+3}{x}$ $g(x) = \frac{2x+3}{5x-2}$

D_f $x \in \mathbb{R}, x \neq 0$ D_g $x \in \mathbb{R}, x \neq \frac{2}{5}$

A) $A(x) = \frac{f(x)}{g(x)}$ 5pts

D_A $x \in \mathbb{R}, x \notin \{0, \frac{2}{5}, \frac{-3}{2}\}$

$g(x) \overset{\text{set}}{=} 0 = \frac{2x+3}{5x-2}$

$x = \frac{-3}{2}$

B) $B(x) = g(f(x))$ 5pts

D_B $x \in \mathbb{R}, x \notin \{0, \frac{-15}{8}\}$

$\frac{2x+3}{x} \overset{\text{set}}{=} \frac{2}{5}$

$10x + 15 = 2x$

$8x = -15$

$x = \frac{-15}{8}$

C) $C(x) = \frac{1}{\frac{1}{f} + \frac{1}{g}}$ 5pts

D_C $x \in \mathbb{R}, x \notin \{\frac{5}{3}, 0, \frac{2}{5}, \frac{1}{3}\}$

$\frac{x}{2x+3} + \frac{5x-2}{2x+3} \overset{\text{set}}{=} 0$

$6x = 2$

$x = \frac{1}{3}$

V For each element of Domain there is exactly one element in the range. Show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

5pts

B) 5pts

$g(x) = \frac{2x+3}{5x-2}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$f^{-1} = z = \frac{2x+3}{5x-2}$

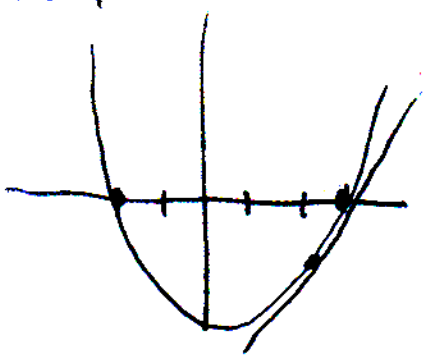
$f^{-1} = f$

$f(f^{-1}(x)) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2}$

$= \frac{\frac{4x+6+15x-6}{5x-2}}{\frac{10x+15-10x+4}{5x-2}} = \frac{19x}{19} = x.$

QED

Extra.



$f(x) = (x+2)(x-3)$

$f(2) = -4$

$m = 3$

$(2, -4)$

Wrong

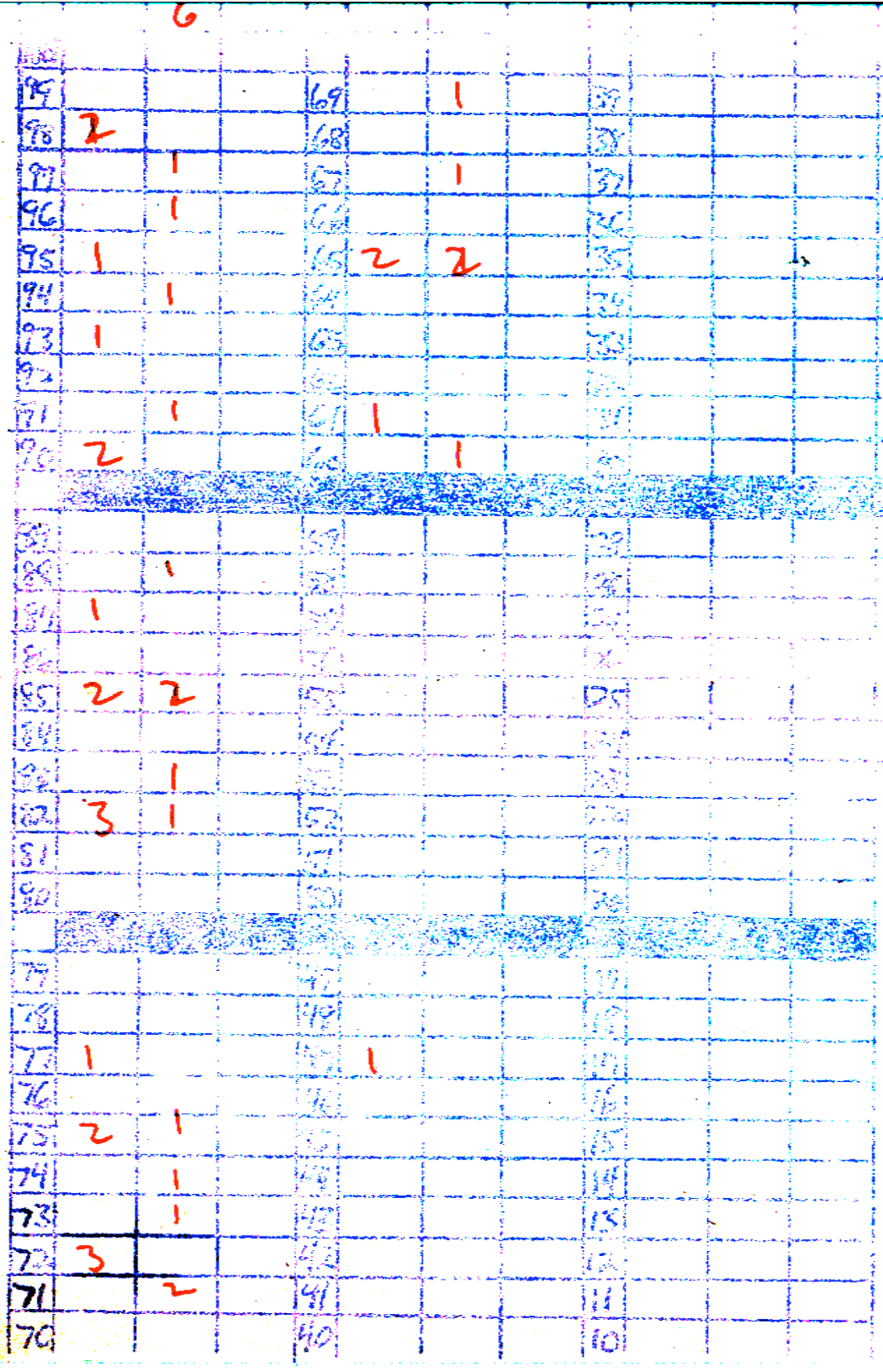
$y + 4 = 2(x-3)$

$y = 2x - 10$

$0 = 2x - 10$

$x = 5$

x intercept is $(5, 0)$



100's	0	0	0
70's	6	4	10
80's	6	5	11
70's	6	5	11
60's	3	5	8
50's	1	0	1
Total	22	19	41

Total Points	1748	1501	
Total Attempts	22	19	41
Accuracy	79.5	79.0	79.24

Excellent exam — Nobody could or had time to do extra. No need to change. most wasted time doing unnecessary stuff on Domain only stuff — make changes as shown on key.

Class: ANALYSIS
Chapter: 1
Pages:
DATE: 2/7/82

ANALYSIS CHAPTER 1

NAME Key

Per 1982

I State the DOMAIN ONLY for each of the following. (5 pts ea)

A) $A(x) = \dfrac{x\sqrt{x}}{x^2 + 5x + 6}$ B) $B(x) = \dfrac{\sqrt{x+3}}{x^2 + x}$ C) $C(x) = \sqrt{x^2 + 6x + 12}$

II Graph each of the following. Carefully ~~show the analysis process at work.~~ (10 pts ea) 5

A) $A(x) = \dfrac{|x+2|}{(x-1)(x+2)}$ B) $B(x) = \sqrt{\dfrac{1}{x} - 1}$ C) $C(x) = 2^{[x]} + 1$ $(-3 \le x \le 2)$

for each of the following!

III ~~Graph and~~ re-define as a "piece-meal function" (10 pts ea) ← explain the meaning of this better during lectures

then GRAPH each

A) $A(x) = x\,Sgn(x)$ B) $B(x) = (Sgn(x))^2 + Sgn(x)$ C) $C(x) = \dfrac{x}{|x|}$

IV Given $f(x) = \dfrac{2x+3}{x}$ $g(x) = \dfrac{2x+3}{5x-2}$ (6 pts ea) 10

State the domain only for :

A) $A(x) = \dfrac{f(x)}{g(x)}$ B) $B(x) = g[f(x)]$ C) $C(x) = \dfrac{1}{\frac{1}{f(x)} + \frac{1}{g(x)}}$

V (10 pts total)

A) Explain what is meant by a 1 : 1 function. What is necessary to prove a function is 1 : 1 ?

B) Prove the function g(x) mentioned in part IV above is or is not a 1 : 1 function.

EXTRA CREDIT

A line is tangent to $f(x) = (x+2)(x-3)$ at $x = 2$. The slope of the tangent line is 3. What are the coordinates of the x-intercept of that tangent line?

Also show idea of $L(x)$ on Ditto photos.
ie $x \to \infty$ $f(x) \to x+1$
etc.

I A) $A(x) = \sqrt{9-x^2}$

D_A $|x| \le 3$

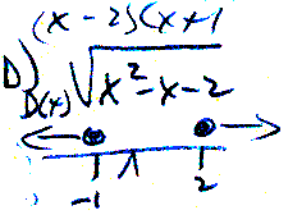
B) $B(x) = \sqrt{\frac{x}{x-1}}$

D_B $x > 1$ or $x \le 0$

C) $C(x) = \sqrt{25}$

D_C $x \in \mathbb{R}$

D) $D(x) = \sqrt{(x-2)(x+1)}$ or $\sqrt{x^2-x-2}$



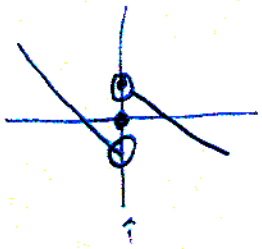
D_D $x \ge 2$ or $x \le -1$

x	x
-x	-x

II A) $A(x) = \operatorname{sgn}^{-1}(x) - x$

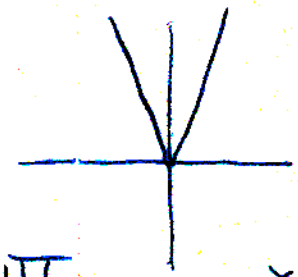
1
0
-1

$A(x) = \begin{cases} 1-x & \text{if } x>0 \\ 0 & \text{if } x=0 \\ -1-x & \text{if } x<0 \end{cases}$



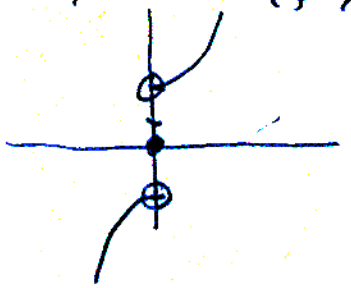
B) $B(x) = |x| + x\operatorname{sgn}(x)$

$B(x) = \begin{cases} 2x & \text{if } x \ge 0 \\ -2x & \text{if } x<0 \end{cases} = 2|x|$



C) $C(x) = x|x| + 2\operatorname{sgn}(x)$

$C(x) = \begin{cases} x^2+2 & \text{if } x>0 \\ 0 & \text{if } x=0 \\ -x^2-2 & \text{if } x<0 \end{cases}$



III $f(x) = \frac{x+2}{x^2+5x+6} = \frac{x+2}{(x+2)(x+3)}$

D_f $x \ne \{-2, -3\}$

$G(x) = \frac{(2x-1)^2}{x^2-3x+2} = \frac{(2x-1)^2}{(x-1)(x-2)}$

D_g $x \ne \{1, 2\}$

A) $A(x) = \frac{f}{g}$

D_A $x \ne \{-2, -3, 1, 2, \frac{1}{2}\}$

B) $B(x) = G(f(x))$

D_B $x \ne \{-2, -3, -\frac{5}{2}\}$

C) $C(x) = \sqrt{\frac{1}{f(x)}}$ $x > -2$ or -2

D_C $x > -3$, $x \ne -2$

$\frac{1}{x+3} > 0$

$x+3 > 0$

$x > -3$

D) $D(x) = G(f(x))$

D_D $x \ne \{-2, -3, -\frac{7}{2}, -\frac{10}{3}\}$

$\frac{1}{x+3} \overset{\text{cut}}{=} -2$

$1 = -2x-6$

$2x = -7$

$x = -\frac{7}{2}$

$\frac{1}{x+3} \overset{\text{cut}}{=} -3$

$1 = -3x-9$

$3x = -10$

$x = -\frac{10}{3}$

$\frac{1}{x+3} \overset{\text{cut}}{=} 1$

$1 = x+3$ $x = -2$

$\frac{1}{x+3} \overset{\text{cut}}{=} 2$

$1 = 2x+6$

$2x = -5$

$x = -\frac{5}{2}$

10 $f(x) = \frac{2x+3}{5x-2}$ $g(x) = \frac{2x+3}{x}$

A D_f $x \neq \frac{2}{5}$ B) D_g $x \neq 0$

5pts 5pts

$x = \frac{2z+3}{5z-2}$

$5xz - 2z = 2x + 3$

$z = \frac{2x+3}{5x-2}$

R_f $f(x) \neq \frac{2}{5}$

$x = \frac{3z+3}{z}$

$xz - 3z = 3$

$z = \frac{3}{x-3}$

R_g $g(x) \neq 3$

c) $f(f(x)) =$

$$\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2} =$$

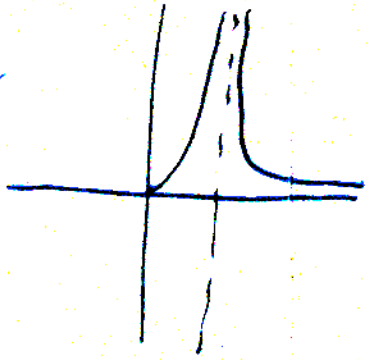
$$\frac{\frac{4x+6+15x-6}{5x-2}}{\frac{10x+15-10x+4}{5x-2}}$$

$\frac{19x}{19} = x$ 10pts

f is 1:1.

10pts

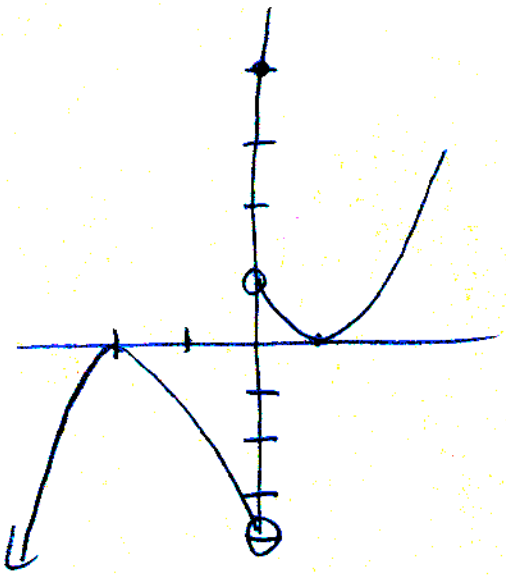
11 $A(x) = \sqrt{\frac{x}{(x-1)^2}}$



D_A $x > 0, x \neq 1$

R_A $A(x) \geq 0$

EXTRA



$$y = \begin{cases} (x-1)^2 & x > 0 \\ 4 & x = 0 \\ -(x+2)^2 & x < 0 \end{cases}$$

| | | |

ANALYSIS Chapter 1 NAME Key 1983

SHOW ALL WORK CLEARLY AND CAREFULLY PER ch 1

I whole section on final. State the **Domain ONLY** for each of the following. (5 pts ea)

A) $A(x) = \sqrt{9 - x^2}$ B) $B(x) = \sqrt{\frac{x}{x - 1}}$ C) $C(x) = \sqrt{25}$ D) $D(x) = \sqrt{x^2 - x - 2}$

II whole section on final For each of the following, re-define as a "piece-meal" function. Sketch a graph. (10 pts ea)

A) $A(x) = Sgn(x) - x$ B) $B(x) = |x| + x\,Sgn(x)$ C) $C(x) = x|x| + 2Sgn(x)$

III Given $F(x) = \frac{x + 2}{x^2 + 5x + 6}$ and $G(x) = \frac{(2x - 1)}{x^2 - 3x + 2}$

Determine the DOMAIN **ONLY** for each of the following: (5 pts ea)

A) $A(x) = \frac{F(x)}{G(x)}$ B) $B(x) = G(F(x))$ C) $C(x) = \sqrt{\frac{1}{F(x)}}$ D) $D(x) = F(G(x))$

IV Given $F(x) = \frac{2x + 3}{5x - 2}$ and $G(x) = \frac{2x + 3}{x}$ 20 pts.

A) Find the Domain and Range of F(x) use "inverse" method to determine the range. (5 pts)

B) Find the Domain and Range of G(x) use (5 pts)

C) Prove F(x) is or is not a **1 : 1** function. (10 pts) 5

V ~~State the Domain and Range and sketch a graph of~~ $A(x) = \sqrt{\frac{x}{(x - 1)^2}}$ (10 pts)

Bonus

EXTRA CREDIT ----------

SKETCH a graph of : $Y = \begin{cases} (x - 1)^2 & \text{if } x > 0 \\ 4 & \text{if } x = 0 \\ -(x + 2)^2 & \text{if } x < 0 \end{cases}$

I 5pts ea.

A) $A(x) = \dfrac{\sqrt{x+3}}{x^2+x}$, $x(x+1)$

D_A $x \geq -3$, $x \neq \{0, -1\}$ 2pts

B) $B(x) = \sqrt{\frac{1}{x} - x}$, $\frac{1-x^2}{x^2}$

$+ \circ - \circ + \circ -$ (at -1, 0, 1)

D_B $x \leq -1$ or $0 < x \leq 1$

C) $C(x) = \dfrac{4(x+1)}{(x-1)(x+1)}$ 2pts

D_C $x \neq \{\pm 1\}$

D) $D(x) = \sqrt{-(x+1)(x-3)}$

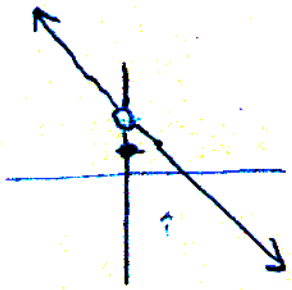
$- \circ + \circ -$ (at -1, 3)

D_D $-1 \leq x \leq 3$

II 10pts ea

A) $A(x) = (\operatorname{Sgn}(x))^2 - x + 1$

$$A(x) = \begin{cases} -x+2 & \text{if } x > 0 \\ 1 & \text{if } x = 0 \\ -x+2 & \text{if } x < 0 \end{cases}$$

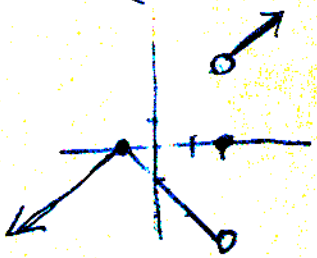


B) $B(x) = |x+1| \operatorname{Sgn}(x-2)$

$x+1 = -x-1$, $-x-1 = -x+1$

$\circ$ (at -1) $\circ$ (at 2)

$$B(x) = \begin{cases} 0 & x \in \{-1, 2\} \\ x+1 & x > 2 \\ -x-1 & -1 < x < 2 \\ x+1 & x < -1 \end{cases}$$



C) $C(x) = |x+1| + |x-1|$

$\circ$ (at -1) $\circ$ (at 1)

$$C(x) = \begin{cases} 2x & \text{if } x \geq 1 \\ 2 & \text{if } -1 \leq x < 1 \\ -2x & \text{if } x < -1 \end{cases}$$

III $A(x) = 2^{[x]} + 1$

D_A $-2 \leq x \leq 2$

$$A(x) = \begin{cases} 5 & x = 2 \\ 3 & 1 \leq x < 2 \\ 2 & 0 \leq x < 1 \\ \frac{3}{2} & -1 \leq x < 0 \\ \frac{5}{4} & -2 \leq x < -1 \end{cases}$$

IV $f(x) = \dfrac{x-2}{x^2-5x+6}$, $(x-2)(x-3)$ $\quad g(x) = \dfrac{x+3}{x^2+3x+2}$, $(x+2)(x+1)$

5pts ea

A) $A(x) = \dfrac{f(x)}{g(x)}$

D_A $x \neq \{-1, -2, -3, 2, 3\}$

B) $B(x) = G(f(x))$

$\frac{1}{x-3} \overset{\text{Sat}}{\geq} -2$ $\quad \frac{1}{x-3} \overset{\text{Sat}}{\geq} -1$

$1 \geq -2x+6$ $\quad 1 \geq -x+3$

$2x \geq 5$ $\quad x \geq 2$

$x \geq \frac{5}{2}$

D_B $x \neq \{2, 3, \frac{5}{2}\}$

C) $C(x) = \sqrt{\frac{1}{f(x)}}$

$x - 3 \overset{\text{Sat}}{\geq} 0$

$-x \geq +3$

D_C $x > 3$

D) $D(x) = f(f(x))$

$\frac{1}{x-3} \overset{\text{Sat}}{\geq} 2$ $\quad \frac{1}{x-3} \overset{\text{Sat}}{\geq} 3$

$1 \geq 2x-6$ $\quad 1 \geq 3x-9$

$2x \geq 7$ $\quad 3x \geq 10$

$x \geq \frac{7}{2}$ $\quad x \geq \frac{10}{3}$

D_D $x \neq \{2, 3, \frac{7}{2}, \frac{10}{3}\}$

"Extras" -1 ea.

U 20pts total

$f(x) = \frac{2x+3}{5x-2}$ $\quad g(x) = \frac{2x+3}{x}$

a) $x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2}$

$D_f \quad x \neq \frac{2}{5}$

$R_f \quad f(x) \neq \frac{2}{5}$

b) $x = \frac{2z+3}{z}$

$xz - 2z = 3$

$z = \frac{3}{x-2}$

$D_g \quad x \neq 0$

$R_g \quad G(x) \neq 2$

c) f is 1:1 if $f^{-1}(f(x)) = x$

"not a proof" -5

$f\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2} =$

$\frac{\frac{4x+6+15x-6}{5x-2}}{\frac{10x+15-10x+4}{5x-2}} = \frac{19x}{19} = x$

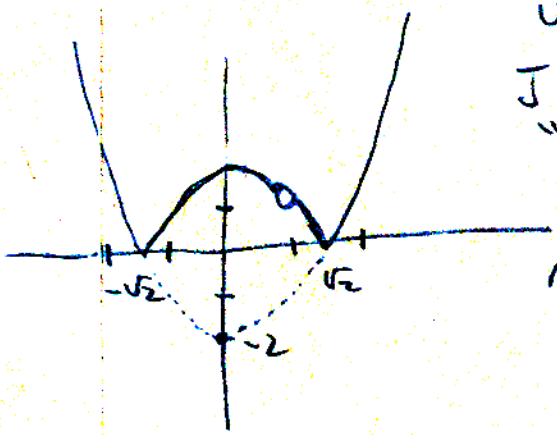
∴ f is a 1:1 f ✓N.

Extra:

$$\begin{array}{r} x^2 - 2 \\ x-1 \overline{)\, x^3 - x^2 - 2x + 2} \\ \underline{x^3 - x^2} \\ -2x \\ \underline{-2x + 2} \\ 0 \end{array}$$

$y = \left| \frac{x^3 - x^2 - 2x + 2}{x-1} \right|$

$\therefore y = \left| \frac{(x^2-2)(x-1)}{x-1} \right| = |x^2 - 2|$



a parabola vertex at -2

Just "fold up" the negative part -

Students need to graph simple 1st degree equations in two. But use intervals [-3, 1] and [2, 4] etc.

Make a ditto & add one HW assignment of this.

---

tests graded pretty hard — but students allowed to earn up to ½ of pts lost depending on completeness and correctness of ex sum corrections.

2/16/84

	6	5	T
100	0	0	0
90	3	2	5
80	3	6	9
70	11	9	20
60	4	6	10
54	2	3	5
	23	26	49

1707 1907 3614

74.2 73.3 73.8

2/17/89

	2	5	T
102 ↓ 51			
100	1	0	1
90	7	6	13
80	4	4	8
70	7	5	12
60	1	0	1
54	2	0	2
	82.09	85.73	83.57

make change as shown I goofed easy because of my omission on Part II generally - I can attempt Earlier Idea (at top of page) seems like good idea — I don't know where to add it, but it should be done!

Honors ANALYSIS

NAME ANALYSIS ch 1 1989

Convince ME that *YOU* know the concept!

PER _____ DATE _____

Chapter 1 EXAM

I State the DOMAIN *ONLY* for each of the following: (5 pts ea)

A) $A(x) = \dfrac{\sqrt{x+3}}{x^2+x}$ B) $B(x) = \sqrt{\dfrac{1}{x} - x}$ C) $C(x) = \dfrac{4(x+1)}{(x-1)(x+1)}$ D) $D(x) = \sqrt{-(x+1)(x-3)}$

II For each of the following, re-define as a "piece-meal" function. Sketch a graph. (10 pts ea)

A) $A(x) = (\text{Sgn}(x))^2 - x + 1$ B) $B(x) = |x+1|\,\text{Sgn}(x-2)$ C) $C(x) = |x+1| + |x-1|$

III Re-define as a "piece-meal" function. Sketch a graph: $A(x) = 2^{[x]} + 1$; $D_A\ -2 \le x \le 2$ (10 pts)

Let $[x]$ mean the Greatest integer of x

IV Given: $F(x) = \dfrac{x-2}{x^2-5x+6}$ and $G(x) = \dfrac{x+3}{x^2+3x+2}$ (5 pts ea)

Determine the DOMAIN ONLY for each of the following:

A) $A(x) = \dfrac{F(x)}{G(x)}$ B) $B(x) = G(F(x))$ C) $C(x) = \sqrt{\dfrac{1}{F(x)}}$ D) $D(x) = F(F(x))$

V Given: $F(x) = \dfrac{2x+3}{5x-2}$ and $G(x) = \dfrac{2x+3}{x}$ (20 pts total)

A) Using the "inverse method", find the Range of F(x).

B) Using the "inverse method", find the Range of G(x).

C) PROVE F(x) is or is not a 1 : 1 function

EXTRA CREDIT 5 PTS

Sketch a graph of: $Y = \left| \dfrac{x^3 - x^2 - 2x + 2}{x-1} \right|$

I

A) $A(x) = \dfrac{x\sqrt{x}}{x^2-5x+6}$

$(x-2)(x-3)$

D_A $x \geq 0$, $x \neq \{2,3\}$

B) $\sqrt{\frac{1}{x}-1}$

$\frac{1-x}{x}$

$-$ 0 $+$ 0 $-$

0 1

D_B $0 < x \leq 1$

$x \neq 1$ -3

C) $\sqrt{x^2+2x+12}$

$x = \frac{-2 \pm \sqrt{4-48}}{2}$ neg under root → complex

$(x+1)^2+11$

$\therefore \geq 11$ So $x \in \mathbb{R}$

OR

Show "Does it cross" Set its value > 0 or < 0

$x=0 \Rightarrow 12 \therefore A(1) > 0$

D) $\sqrt{-(x+2)(x-3)}$

$-$ 0 $+$ 0 $-$

-2 0 3

D_D $-2 \leq x \leq 3$

-3 opposite

II

A) $A(x) = \dfrac{3x+2}{2x-1}$

D_A $x \neq \frac{1}{2}$ R_A $A(x) \neq \frac{3}{2}$

$x = \dfrac{3z+2}{2z-1}$

$2xz - x = 3z + 2$

$(2x-3)z = x+2$

$z = \dfrac{x+2}{2x-3}$

B) $B(x) = \sqrt{2^x}$

D_B $x \in \mathbb{R}$

R_B $B(x) \geq 5$

C) $C(x) = -x$

D_C $x \in \mathbb{R}$

R_C $C(x) \in \mathbb{R}$

III

A $A(x) = |x+2| + \operatorname{Sgn}(x-1)$

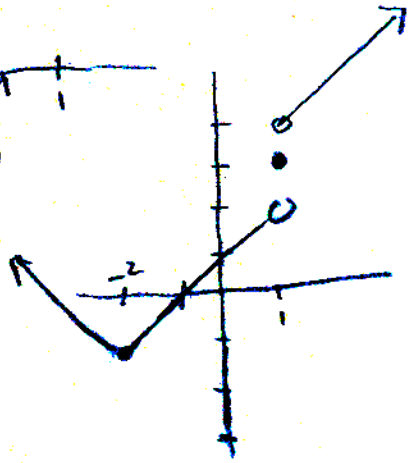
-2 1

$x+2+1$

$x+2-1$

$-x-2-1$

$$A(x) = \begin{cases} x+3, & x > 1 \\ 3, & x = 1 \\ x+1, & -2 \leq x < 1 \\ -x-3, & x < -2 \end{cases}$$



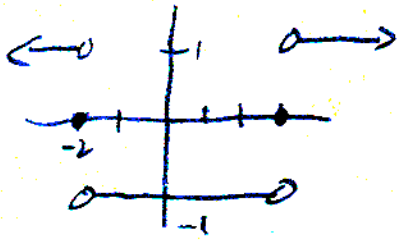
III B)

$B(x) = (\operatorname{Sgn}(x+2))(\operatorname{Sgn}(x-3))$

$+$ 0 $-$ 0 $+$

-2 0 3

$$B(x) = \begin{cases} 1 & x > 3 \text{ OR } x < -2 \\ -1 & -2 < x < 3 \\ 0 & x \in \{-2, 3\} \end{cases}$$

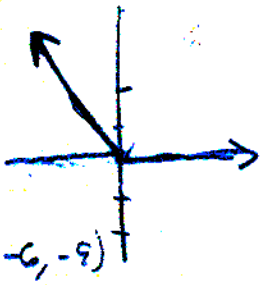


III C)

$C(x) = |x - 1x1|$

$x + x$

$$C(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$



IV

A) $A(0,+1)$ $B(-3,0)$ $C(9,1)$ $D(-6,-9)$

$m_{AB} = \dfrac{-2-0}{0+3} = \dfrac{2}{3}$ $m_{CD} = \dfrac{-9-1}{-6-9} = \dfrac{-10}{-15} = \dfrac{2}{3}$

AB // CD Because Both have = Slopes.

B) $d = \sqrt{(9+3)^2 + 1^2}$

$d = \sqrt{145}$

V $f(x) = \frac{2x+3}{x}$ $g(x) = \frac{2x+3}{5x-2}$

D_f $x \neq 0$ D_g $x \neq \frac{2}{5}$

A) $A(x) = \frac{f}{g}$ $g \overset{\text{set}}{=} 0 \Rightarrow x = \frac{-3}{2}$

D_A $x \neq \{0, \frac{2}{5}, \frac{-3}{2}\}$

B) $B(x) = G(f(x))$

D_B $x \neq \{0, \frac{-15}{8}\}$

$\frac{2x+3}{x} \overset{\text{set}}{=} \frac{2}{5}$

$10x+15 = 2x$

$8x = -15$

$x = \frac{-15}{8}$

C) $c(x) = \frac{1}{\frac{1}{f} + \frac{1}{g}}$

D_C $x \neq \{0, \frac{-3}{2}, \frac{2}{5}, \frac{1}{3}\}$

$\frac{1}{f} + \frac{1}{g} \overset{\text{set}}{=} 0$

$\frac{x}{2x+3} + \frac{5x-2}{2x+3} = 0$

$6x - 2 = 0$

$x = \frac{1}{3}$

3 pts

D) $D(x) = G(G(x))$

D_x $x \neq \{\frac{2}{5}\}$

$\frac{2x+3}{5x-2} \overset{\text{set}}{=} \frac{2}{5}$

$10x+15 = 10x-4$

$15 = -4$

no additional restrictions

2 pts

VI

A) G^{-1}

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3x$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2} = G^{-1}$

B) $(G)^{-1} = \frac{5x-2}{2x+3}$

VI c

C) $f(g(x))$

$f\left(\frac{2x+3}{5x-2}\right) = \frac{2\left(\frac{2x+3}{5x-2}\right)+3}{\frac{2x+3}{5x-2}}$

$= \frac{\frac{4x+6+15x-6}{5x-2}}{\frac{2x+3}{5x-2}}$

$= \frac{\frac{19x}{5x-2}}{\frac{2x+3}{5x-2}} = \frac{19x}{2x+3}$

EXTRA credit

$f(x) = (x+2)(x-3)$

$f(2) = (4)(-1) = -4$

$(2, -4)$ $m = 3$

$y+4 = 3(x-2)$

$y = 3x - 10$

$0 \overset{\text{set}}{=} 3x - 10$

$x = \frac{10}{3}$.

	5	6	T
100↑	0	1	1
90↑	3	4	7
80↑	9	9	18
70↑	13	6	19
60↑	3	3	6
54↓	2	0	2
	77.17	80.61	78.7

100

51

	2	5	T
100	0	2	2
90	10	7	17
80	7	10	17
70	5	1	6
60	1	1	2
54	0	0	0
	85.7	88.5	87.0

105

68

Still a great exam!

ANALYSIS Chapter 1 NAME 2/8/85 Ch 1 ANALYSIS 2/12/88

SHOW ALL WORK CLEARLY AND CAREFULLY

BE SURE YOU HAVE COPIED THE ORIGINAL PROBLEM CORRECTLY !! PER [redacted]

I State the **Domain ONLY** for each of the following. (5 pts ea)

A) $A(X) = \dfrac{X\sqrt{X}}{X^2 - 5X + 6}$ B) $B(x) = \sqrt{\frac{1}{X} - 1}$ C) $C(X) = \sqrt{X^2 + 2X + 12}$ D) $D(X) = \sqrt{-(X + 2)(X - 3)}$

II State the DOMAIN AND RANGE for each of the following (5 pts ea)

A) $A(X) = \dfrac{2X + 2}{2X - 1}$ B) $B(X) = \sqrt{25}$ C) $C(X) = -X$

III Restate as "piece-meal" functions. SKETCH a graph of each on a seperate, labeled axis. (5 pts ea)

A) $A(X) = |X + 2| + Sgn(X-1)$ B) $B(X) = \Big(Sgn(X+2)\Big)\Big(Sgn(X-3)\Big)$ C) $C(X) = \Big|X - |X|\Big|$

IV Given points A(0,2), B(-3,0), C(9,1), D(-6,-9) (5 pts ea)

A) Prove line segment AB is parallel to line segment CD.

B) Find the length of line segment BC.

V Given $F(X) = \dfrac{2X + 3}{X}$ and $G(X) = \dfrac{2X + 3}{5X - 2}$

STATE **DOMAIN ONLY** FOR — (5 pts ea)

A) $A(X) = \dfrac{F(X)}{G(X)}$ B) $B(X) = G(F(X))$ C) $C(X) = \dfrac{1}{\frac{1}{F(X)} + \frac{1}{G(X)}}$ D) $D(X) = G(G(X))$

VI Using F and G as defined in Part **V**, Find the formula for each of the following (5 pts ea)

DO NOT include Domain information.

A) $A(X) = G^{-1}(X)$ B) $B(X) = (G(X))^{-1}$ C) $D(x) = F(G(X))$

EXTRA CREDIT ———— 5 PTS ————

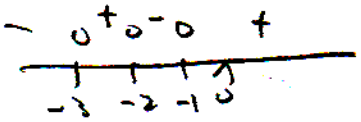
A line is tangent to $F(X) = (X + 2)(X - 3)$ at $X = 2$. The slope of the tangent line is 3. What are the coordinates of the X-intercept of that tangent line?

I

A) $A(x)=\sqrt{\frac{x^2+5x+6}{x+1}}$

5 pts ea

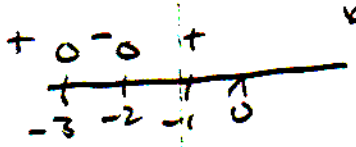
$\frac{(x+2)(x+3)}{x+1}$



D_A $-3 \le x \le -2$ or $x > -1$

B) $B(x)=\frac{\sqrt{x^2+5x+6}}{x+1}$

$\frac{(x+2)(x+3)}{x+1}$



D_B $x \le -3$ or $x \ge -2$, $x \ne -1$

no # lim −3 # lim wrong −2

C) $C(x)=\frac{\sqrt{x-2}}{x^2-4x}$

$x(x-4)$

D_C $x > 2$, $x \ne 4$

D) $D(x)=\sqrt{-x}$

D_D $x \le 0$

II 5 pts ea

A) $A(x)=\frac{3x-2}{6x-4}$

$=\frac{3x-2}{2(3x-2)}$

$=\frac{1}{2}$

R $A(x) \ne \frac{1}{2}$ (−2)

D_A $x \ne \frac{3}{2}$

R_A $A(x) \ne \frac{1}{2}$

B) $B(x)=\frac{3x-2}{5x-4}$

$x=\frac{3z-2}{5z-4}$

$5xz-4x=3z-2$

$z=\frac{4x-2}{5x-3}$

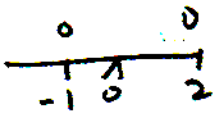
D_B $x \ne \frac{4}{5}$

R_B $B(x) \ne \frac{3}{5}$

C) $C(x)=|2x+1|-3$

D_C $x \in \mathbb{R}$

R_C $C(x) \ge -3$



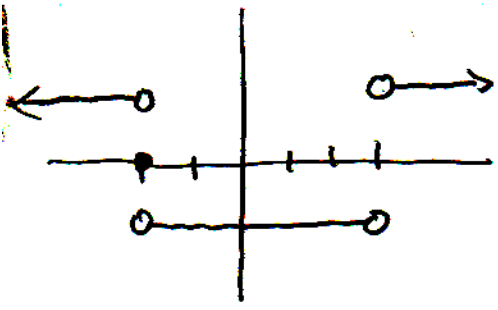
III 5 pts ea

A) $A(x)=\frac{\operatorname{Sgn}(x+2)}{\operatorname{Sgn}(x-3)}$

$\frac{-1}{-1}$

$$A(x)=\begin{cases}1 & \text{if } x<-2\\ 0 & \text{if } x=-2\\ -1 & \text{if } -2<x<3\\ 1 & \text{if } x>3\end{cases}$$

$\frac{1}{-1}$



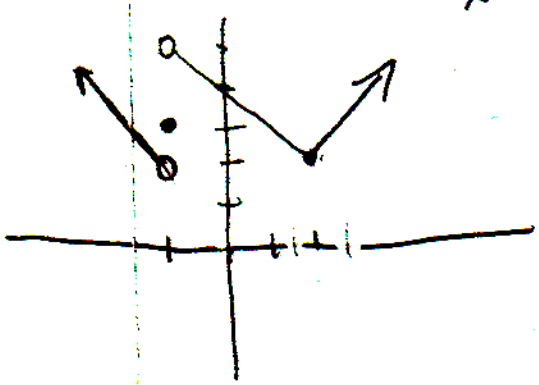
B) $B(x)=|x-2|+2\operatorname{Sgn}(x+1)$

$-x+2+2$

3

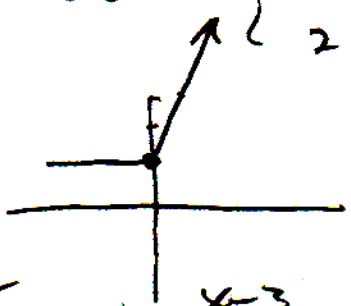
$-x$

$$B(x)=\begin{cases}-x & \text{if } x<-1\\ 3 & \text{if } x=-1\\ -x+4 & \text{if } -1<x<2\\ 2 & \text{if } x=2\\ x & \text{if } x>2\end{cases}$$



$C(x) = x + |x| + 1$

$$C(x) = \begin{cases} 1 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ 2x+1 & \text{if } x > 0 \end{cases}$$



V $f(x) = \dfrac{x-3}{(x-3)^2} = \dfrac{1}{x-3}$ $g(x) = \dfrac{x+3}{(x^2+3x+3)(x+1)(x+2)}$

D_f $x \neq 3$ 5 pts ea

D_g $x \neq \{-1, -2\}$.

4) $q(x) = \dfrac{f(x)}{f(x)}$

D_q $x \neq \{-1, -2, 3\}$

B) $h(x) = f(f(x))$

D_h $x \neq \{3, 2, \frac{5}{2}\}$

$\frac{1}{x-3} \overset{\text{cut}}{=} -1$

$1 = -x + 3$

$x = 2$

$\frac{1}{x-3} \overset{\text{cut}}{=} -2$

$1 = -2x + 6$

$2x = 5$

$x = \frac{5}{2}$

D) $D(x) = f(f(x))$

$\frac{1}{x-3} \overset{\text{cut}}{=} 3$ D_D $x \neq \{3, \frac{10}{3}\}$

$1 = 3x - 9$

$3x = 10$

$x = \frac{10}{3}$

VI $f(x) = \dfrac{x}{2x-3}$

$g(x) = \dfrac{2x-1}{3x-2}$

4) $G(f(x)) = \dfrac{2\left(\frac{x}{2x-3}\right)-1}{3\left(\frac{x}{2x-3}\right)-2}$

$= \dfrac{2x - 2x + 3}{3x - 4x + 6}$

10 pts $= \dfrac{3}{6-x}$

B) $x = \dfrac{z-1}{2z-3}$

$2zx - z = 3x$

$z = \dfrac{3x}{2x-1}$

$f(f(x)) = \dfrac{\frac{3x}{2x-1}}{2\left(\frac{3x}{2x-1}\right)-3}$

$= \dfrac{\frac{3x}{2x-1}}{\frac{6x-6x+3}{2x-1}}$

$= \dfrac{3x}{3}$

$= x$

cont.

III 5 pts ea

A(1, 6) B(-4, -4) C(4, ?) D(3, 5)

1) $AB \parallel CD$ $\dfrac{7-5}{4-3} = 2$ $\parallel$ slopes =.

$\dfrac{6+4}{1+4} = 2$

3) $AB \perp AD$ $\dfrac{6-5}{1-3} = \dfrac{1}{-2}$

$2 \cdot \left(\frac{-1}{2}\right) = -1$ ✓, $\perp$

C) $c(x) = \sqrt{\dfrac{1}{f(x)}}$ $\dfrac{- \; 0 \; + \; 0 \; - \; 0 \; +}{+ \; + \; + \; + \; + \; +}$ $-3 \; -2 \; -1 \; 0$

D_c $-3 \leq x < -2$ OR $x > -1$

extra

$A(x) = \sqrt{2x - 12 - x^2}$

$= \sqrt{-(x^2 - 2x + _) - 12}$

$A(x) = \sqrt{-(x-1)^2 - 13}$

since $-(x-1)^2 \leq 0$

$-(x-1)^2 - 13 \leq -13$

D_A $x \in \{\}$.

they Need longer than 2 weeks
for this units

Make little of Several
Piecemeal tests to do
also more # line => Domain
Problems.

Simple to include Power 1:1 of a few

	2	5	T	
100	2	0	2	101
90	1	4	5	
80	5	2	7	
70	1	8	9	
60	8	4	12	0
59	6	4	10	
	70.5	72.7	71.5	

ANALYSIS Chapter 1 NAME Analysis Ch 1

SHOW ALL WORK CLEARLY AND CAREFULLY

PER ____ 2/7/86 92

BE SURE YOU HAVE COPIED THE ORIGINAL PROBLEM CORRECTLY !!

I State the **Domain ONLY** for each of the following. (5 pts ea)

A) $A(X) = \sqrt{\frac{X^2 + 5X + 6}{X + 1}}$ B) $B(X) = \frac{\sqrt{X^2 + 5X + 6}}{X + 1}$ C) $C(X) = \frac{\sqrt{X - 2}}{X^2 - 4X}$ D) $D(X) = \sqrt{-X}$

II State the DOMAIN AND RANGE for each of the following (5 pts ea)

A) $A(X) = \frac{3X - 2}{6X - 4}$ B) $B(X) = \frac{3X - 2}{5X - 4}$ C) $C(X) = |2X + 1| - 3$

III Restate as "piece-meal" functions. SKETCH a graph of each on a seperate, labeled axis. (5 pts ea) 10

A) $A(X) = \frac{Sgn(X + 2)}{Sgn(X - 3)}$ B) $B(X) = |X - 2| + 2\,Sgn(X + 1)$ C) $C(X) = X + |X| + 1$

IV Given points A(1,6), B(-4,-4), C(4,7), D(3,5) (5 pts ea) 20

A) Prove line segment AB is parallel to line segment CD.

B) Prove line segment AB is perpendicular to line segment AD.

V Given $F(X) = \frac{X - 3}{X^2 - 6X + 9}$ and $G(X) = \frac{X + 3}{X^2 + 3X + 2}$

STATE **DOMAIN ONLY** FOR (5 pts ea)

A) $A(X) = \frac{G(X)}{F(X)}$ B) $B(X) = G(F(X))$ C) $C(X) = \sqrt{\frac{1}{G(X)}}$ D) $D(X) = F(F(X))$

150 Tot

VI Given $F(X) = \frac{X}{2X - 3}$ and $G(X) = \frac{2X - 1}{3X - 2}$ (10 pts ea)

A) Find the formula ONLY for G(F(X))

B) PROVE F(X) is a 1 : 1 function.

EXTRA CREDIT ———— 5 PTS ————

Given $A(X) = \sqrt{2X - 12 - X^2}$. Find D_A . Explain.

I

A) $A(x) = \sqrt{9-x^2}$

$- \; 0 \; + \; 0 \; -$

$-3 \quad 0 \quad 3$

$D_A \; |x| \le 3$

B) $B(x) = \sqrt{\frac{x}{x-1}}$

$+ \; 0 \; - \; \text{ND} \; +$

$0 \quad 1$

$D_B \; x \le 0 \text{ or } x > 1$

C) $C(x) = \sqrt{25}$

$D_C \; x \in \mathbb{R}$

D) $D(x) = \sqrt{x^2-x-2}$

$(x+1)(x-2)$

$+ \; 0 \; - \; 0 \; +$

$-1 \quad 0 \quad 2$

$D_D \; x \le -1 \text{ or } x \ge 2$

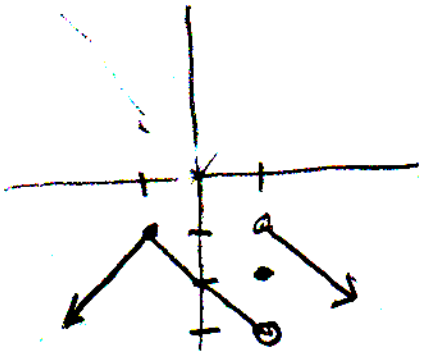
II

A) $A(x) = \text{Sgn}(x-1) - |x+1|$

I II III IV V

$-1 \quad 0 \quad 1$

$$A(x) = \begin{cases} x & \text{if } x < -1 \\ -1 & \text{if } x = -1 \\ -x-2 & \text{if } -1 < x < 1 \\ -2 & \text{if } x = 1 \\ -x & \text{if } x > 1 \end{cases} \qquad \begin{array}{l} -1+x+1 \\ -1 \\ -1-x-1 \\ 0-2 \\ 1-x-1 \end{array}$$

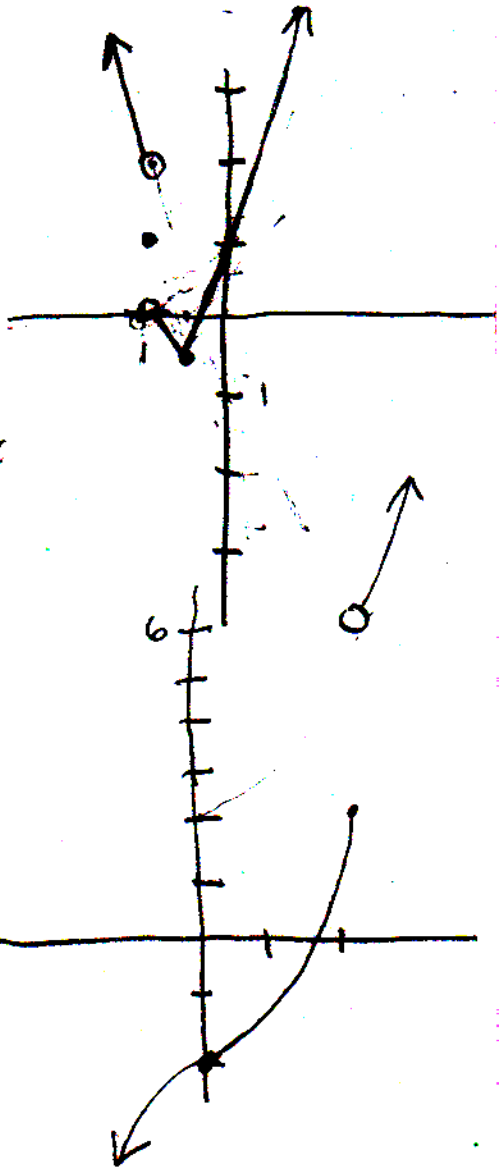


B) $B(x) = |2x+1| + x\,\text{Sgn}(x+1)$

$0 \quad 0$

$-1 \quad -\frac{1}{2} \quad 0$

$$B(x) = \begin{cases} -3x-1 & \text{if } x < -1 \\ 1 & \text{if } x = -1 \\ -x-1 & \text{if } -1 < x < -\frac{1}{2} \\ -\frac{1}{2} & \text{if } x = -\frac{1}{2} \\ 3x+1 & \text{if } x > -\frac{1}{2} \end{cases} \qquad \begin{array}{l} -(2x+1)-x \\ \\ -2x-1+x \\ 0+x \\ 2x+1+x \end{array}$$



C) $C(x) = x|x| + 2\,\text{Sgn}(x-2)$

$0 \quad 0$

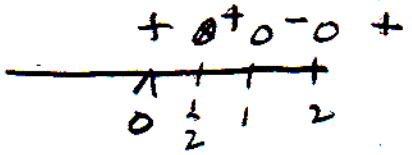
$0 \quad 2$

$$C(x) = \begin{cases} -x^2-2 & \text{if } x < 0 \\ -2 & \text{if } x = 0 \\ x^2-2 & \text{if } 0 < x < 2 \\ 4 & \text{if } x = 2 \\ x^2+2 & \text{if } x > 2 \end{cases} \qquad \begin{array}{l} -x^2-2 \\ 0-2 \\ x^2-2 \\ 4 \\ x^2+2 \end{array}$$

III $f(x)=\dfrac{x+2}{x^2+5x+6}=\dfrac{x+2}{(x+2)(x+3)}$ $G(x)=\dfrac{(2x-1)^2}{x^2-3x+2}=\dfrac{(2x-1)^2}{(x-2)(x-1)}$

D_f $x\neq\{-2,-3\}$ D_G $x\neq\{2,1\}$

C) $\sqrt{\frac{f}{G}}$ G



D_C $x\leq 1,\ x\neq\frac{1}{2}$ OR $x>2$

D) $f(f)$ $\frac{1}{x+3}\overset{SJT}{=}-2$ $\frac{1}{x+3}\overset{SJT}{=}-3$

$1=-2x-6$ $1=-3x-9$

$2x=-7$ $3x=-10$

$x=-\frac{7}{2}$ $x=-\frac{10}{3}$

D_D $x\neq\{-2,-3,-\frac{7}{2},-\frac{10}{3}\}$.

IV

A) $\frac{f}{g}$ D_A $x\neq\{-2,-3,1,2,\frac{1}{2}\}$

B) $G(f)$ D_B $x\neq\{-2,-3,-\frac{5}{2}\}$

$\frac{1}{x+3}\overset{SJT}{=}2$ $\frac{1}{x+3}\overset{SJT}{=}1$

$1=2x+6$ $1=x+3$

$x=-\frac{5}{2}$ $x=-2$

VII

A) $f(x)=\dfrac{2x+3}{5x-2}$

$x=\dfrac{2z+3}{5z-2}$

$5xz-2x=2z+3$

$(5x-2)z=2x+3$

$z=\dfrac{2x+3}{5x-2}$

D_f $x\neq\frac{5}{2}$

R_f $f(x)\neq\frac{5}{2}$

B) $G(x)=\dfrac{2x+3}{x}$

$x=\dfrac{2z+3}{z}$

$xz-2z=3$

$z=\dfrac{3}{x-2}$

D_G $x\neq 0$

R_G $G(x)\neq 2$

C) $f\left(\frac{2x+3}{5x-2}\right)=\dfrac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x-2}\right)-2}=$

$\dfrac{4x+6+15x-6}{10x+15-10x+4}=\dfrac{19x}{19}=x$

$\therefore f$ is 1:1.

V

A) MAB $\dfrac{2-0}{0+3}=\dfrac{2}{3}$ ← $\Rightarrow$ ↓ 1:1

MCD $\dfrac{1+9}{9+6}=\dfrac{10}{15}=\dfrac{2}{3}$

B) $d=\sqrt{(9+3)^2+(1)^2}$

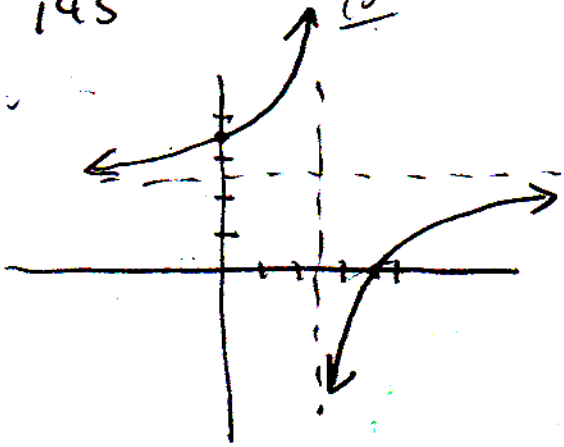
$=\sqrt{145}$

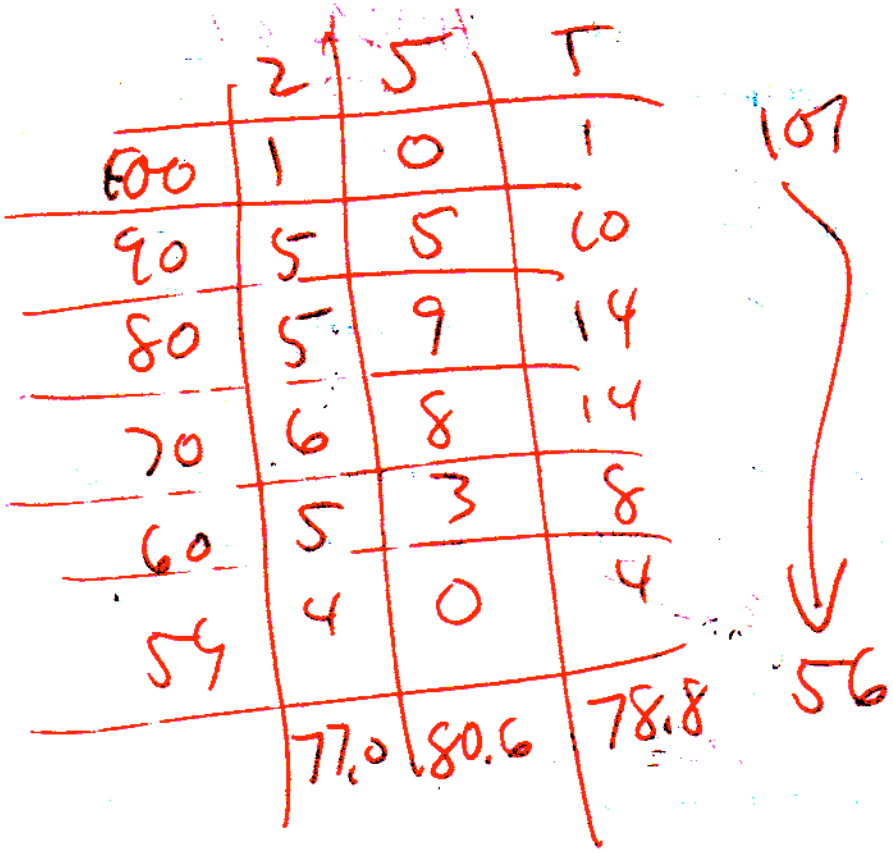
29

145

10

EXTra

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80.6
78.8
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56

ANALYSIS Chapter 1 NAME ANC1 87

SHOW ALL WORK CLEARLY AND CAREFULLY

PER

I State the **Domain ONLY** for each of the following. (5 pts ea)

A) $A(x) = \sqrt{9 - x^2}$ B) $B(x) = \sqrt{\frac{x}{x-1}}$ C) $C(x) = \sqrt{25}$ D) $D(x) = \sqrt{x^2 - x - 2}$

II For each of the following, re-define as a "piece-meal" function. Sketch a graph. (10 pts ea)

A) $A(x) = Sgn(x-1) - |x+1|$ B) $B(x) = |2x + 1| + x\,Sgn(x+1)$ C) $C(x) = x|x| + 2Sgn(x-2)$

III Given $F(x) = \frac{x + 2}{x^2 + 5x + 6}$ and $G(x) = \frac{(2x - 1)^2}{x^2 - 3x + 2}$

Determine the DOMAIN **ONLY** for each of the following: (5 pts ea)

A) $A(x) = \frac{F(x)}{G(x)}$ B) $B(x) = G(F(x))$ C) $C(x) = \sqrt{\frac{1}{G(x)}}$ D) $D(x) = F(F(x))$

IV Given $F(x) = \frac{2x + 3}{5x - 2}$ and $G(x) = \frac{2x + 3}{x}$ (20 pts tot)

A) Using the inverse method, find the Domain and Range of F(x)

B) Using the inverse method, find the Domain and Range of G(x)

C) Prove F(x) is or is not a **1 : 1** function.

V Given points : A(0,2) , B(-3,0) , C(9,1) , D(-6,-9) (5 pts ea)

A) **Prove line segment AB is parallel to line segment CD.**

B) **Find the length of line segment BC.**

EXTRA CREDIT ----------5 pts--

Graph the function F(x) in section **IV**

I A) $A(x)=\sqrt{9-x^2}$

$-$ 0 $+$ 0 $-$ on -3, 0, 3

D_A $|x|\le 3$

B) $B(x)=\sqrt{\frac{x}{x-1}}$

$+$ U/D $-$ 0 $+$ on 0, 1

D_B $x\le 0$ or $x>1$

C) $C(x)=\sqrt{25}$

D_C $x\in\mathbb{R}$

D) $D(x)=\sqrt{x^2-x-2}$

$(x-2)(x+1)$

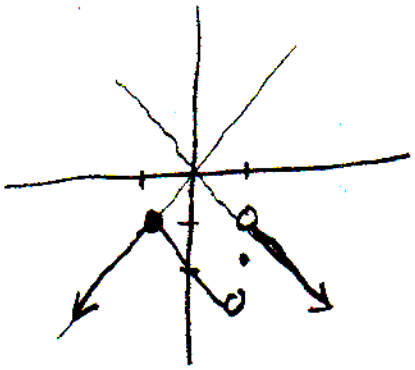
$+$ 0 $-$ 0 $+$ on -1, 2

D_D $x\le -1$ or $x\ge 2$

II A) $A(x)=\operatorname{Sgn}(x-1)-|x+1|$

0 0 on -1, 0, 1

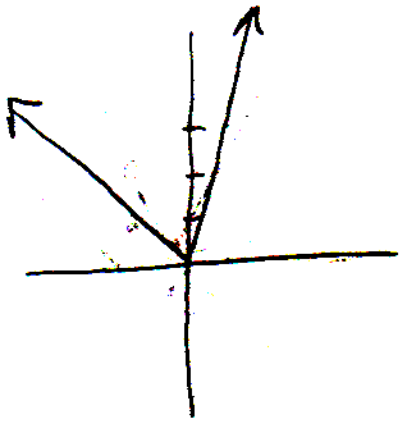
$$A(x)=\begin{cases} x & \text{if } x<-1 \quad -1+x+1 \\ -1 & \text{if } x=-1 \quad -1-0 \\ -2-x & \text{if } -1<x<1 \quad -1-x-1 \\ -2 & \text{if } x=1 \quad 0-2 \\ -x & \text{if } x>1 \quad 1-x-1 \end{cases}$$



B) $B(x)=|x|+x+x\operatorname{Sgn}x$

0 on 0

$$B(x)=\begin{cases} -x & \text{if } x\le 0 \quad -x+x-x \\ 3x & \text{if } x>0 \quad x+x+x \end{cases}$$



III $f(x)=\dfrac{x+2}{x^2+5x+6}=\dfrac{1}{x+3}$ $\quad$ $g(x)=\dfrac{(2x-1)^2}{x^2-3x+2}$

$(x+3)(x+2)$ $\quad$ $(x-2)(x-1)$

D_f $x\neq\{-2,-3\}$ $\quad$ D_g $x\neq\{1,2\}$

4) $A(x)=\dfrac{f}{g}$

D_A $x\neq\{-2,-3,1,2,\frac{1}{2}\}$

$(2x-1)^2\overset{SVT}{=}0$

$x=\frac{1}{2}$

B) $B(x)=G(f(x))$

D_B $x\neq\{-2,-3,-\frac{5}{2}\}$

$\frac{1}{x+3}\overset{SVT}{=}1$ $\quad$ $\frac{1}{x+3}\overset{SVT}{=}2$

$1=x+3$ $\quad$ $1=2x+6$

$x=-2$ $\quad$ $x=-\frac{5}{2}$

C) $C(x)=\sqrt{\frac{1}{G(x)}}$

D_C $x\neq 1$; $x\neq\frac{1}{2}$ or $x>2$

$+$ ∄ $+$ 0 $-$ 0 $+$ on 0, $\frac{1}{2}$, 1, 2

D) $D(x)=f(f(x))$

D_D $x\neq\{-2,-3,-\frac{7}{2},-\frac{10}{3}\}$

$\frac{1}{x+3}\overset{SVT}{=}-2$ $\quad$ $\frac{1}{x+3}\overset{SVT}{=}-3$

$1=-2x-6$ $\quad$ $1=-3x-9$

$-7=x$ $\quad$ $x=-\frac{10}{3}$

IV

4) $f(x) = \frac{2x+3}{5x-2}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2} = f^{-1}(x)$

$D_f \quad x \neq \frac{2}{5}$

$R_f \quad f(x) \neq \frac{2}{5}$

B) $g(x) = \frac{2x+3}{x}$

$x = \frac{2z+3}{z}$

$xz - 2z = 3$

$z = \frac{3}{x-2}$

$D_g \quad x \neq 0$

$R_g \quad g(x) \neq 2$

C) $f\left(\frac{2x+3}{5x-2}\right) =$

$\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x+2}\right)-2}$

$\frac{4x+6+15x-6}{10x+15-10x+4}$

$\frac{19x}{19}$

x $\quad$ f is 1:1

V $\quad A(0,2) \quad B(-3,0) \quad C(-9,1) \quad D(-6,-9)$

4) $AD \parallel CD$

$m_{AB} = \frac{2-0}{0+3} = \frac{2}{3}$

$m_{CD} \quad \frac{1+9}{9+6} = \frac{-16}{15} = \frac{2}{3}$

Slopes $= \therefore \parallel$

B) Length BC

$l = \sqrt{(9+3)^2 + 1^2}$

$= \sqrt{144+1}$

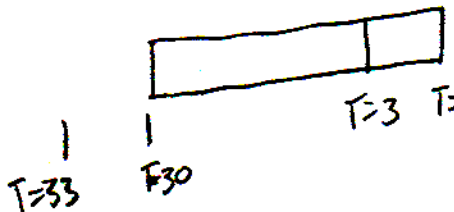
$= \sqrt{145} \quad 5.29$

Extra

$\frac{108 \text{ Km}}{\text{h}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ Sec}}$

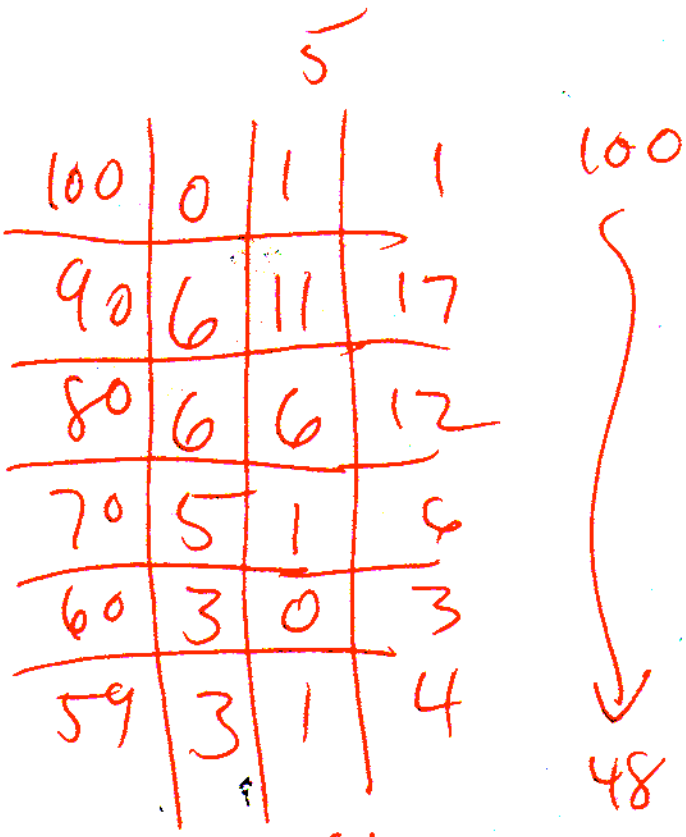
$\frac{1 \text{ Km per sec.}}{36}$

① 3 Sec to pass through entrance of tunnel $\therefore 3 \cdot \frac{1}{36} = \frac{1}{12}$ Km Long.

② 

$\therefore 30 \cdot \frac{1}{36} = \frac{5}{6}$ Km ← The tunnel.

③ without a doubt!



88.1

~~730~~

78.9

83.16

HONORS ANALYSIS
Convince ME that YOU know the concept !
Copy original problem. SHOW ALL WORK !

NAME ANALYSIS Ch 1 1990

PER _____ DATE _____

CHAPTER 1 EXAM

I State the DOMAIN *only* for each of the following: (5 pts ea)

A) $A(x) = \sqrt{9 - x^2}$ B) $B(x) = \sqrt{\frac{x}{x-1}}$ C) $C(x) = \sqrt{25}$ D) $D(x) = \sqrt{x^2 - x - 2}$

II For each of the following, re-define as a " piece-meal " function. SKETCH the graph (15 pts ea)

A) $A(x) = Sgn(x-1) - |x+1|$ B) $B(x) = x + |x| + x\,Sgn(x)$

III Given $F(x) = \frac{x+2}{x^2+5x+6}$ and $G(x) = \frac{(2x-1)^2}{x^2-3x+2}$ (5 pts ea)

Determine the DOMAIN only for each of the following:

A) $A(x) = \frac{F(x)}{G(x)}$ B) $B(x) = G(F(x))$ C) $C(x) = \sqrt{\frac{1}{(G(x)}}$ D) $D(x) = F(F(x))$

IV Given $f(x) = \frac{2x+3}{5x-2}$ and $g(x) = \frac{2x+3}{x}$ (20 pts tot)

A) Using the " inverse method ", find the domain and range of $f(x)$.

B) Using the " inverse method ", find the domain and range of $g(x)$.

C) PROVE $f(x)$ is or is not a " one-to-one" function.

V Given points $A(0, 2)$, $B(-3, 0)$, $C(9, 1)$, $D(-6, -9)$ (5 pts ea)

A) Prove line segment AB is parallel to the line segment CD .

B) Find the length of line segment BC.

EXTRA CREDIT ——— 5 pts ———

A train traveling at 100 km per hour takes three seconds to enter a tunnel and an additional thirty seconds to pass completely through it.

1. What is the length of the train?

2. How long is the tunnel ?

3. Is it true that the more math you have, the harder it is to do arithmetic?

(91)

	2	5	T
100			
90	7	6	13
80	8	9	17
70	5	7	12
60	1	1	2
55	1	1	2

99

44

81,67

1819/22 = 82.7

1938/24 = 80.75

use greatest integer fctn to illustrate "where the fctn changes in its value" and that spot is NOT always at zero

---

Demo to emphasize that x is just x and not |x|

$f(x) = |x+1| + x$

Note that zero is NOT marked on # line.

Emphasize piece wise fctns — one fctns ie they do pass the vertical line test!

Honors Analysis Name AnL ch 1

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date 2003
No Calculators, of course. **CHAPTER #1**

I State the Domain only for each of the following: (5 pts ea)

a) $A(x) = \dfrac{\sqrt{x+3}}{x^2+x}$ b) $B(x) = \sqrt{\dfrac{1}{x} - x}$ c) $C(x) = \dfrac{4(x+1)}{(x-1)(x+1)}$ d) $D(x) = \sqrt{-(x-3)(x+1)}$

II For each of the following, re-define as a "piece-meal" function. Sketch a graph. (10 pts ea)

a) $A(x) = (\text{Sgn}(x))^2 - x + 1$ b) $B(x) = |x+1|\,\text{Sgn}(x-2)$ c) $C(x) = |x+1| + |x-1|$

III Let [x] mean the Greatest Integer Function. Re-define the following as a "piece-meal" function. Graph the function given the domain: D_A $-2 \le x \le 2$. (10 pts)

$A(x) = 2^{[x]} + 1$

IV Given point A with coordinates (2, 2), find the point B (coordinates (x,y)) such that the slope of the line AB is 2 and the distance from point A to point B is 2. (15 pts)

V Given: $F(x) = \dfrac{2x+3}{5x-2}$ and $G(x) = \dfrac{2x+3}{x}$ (5 pts ea)

A) Using the "inverse method", find the range of $F(x)$.

B) Using the "inverse method", find the range of $G(x)$.

C) Prove $F(x)$ is or is not a 1-to-1 function. Specifically state your conclusion.

D) Find the domain of $H(x)$ given: $H(x) = F(G(x))$.

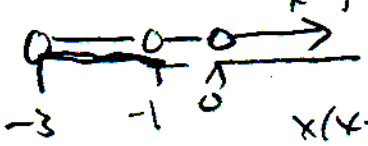
E) Find the domain of $J(x)$ given: $J(x) = \sqrt{\dfrac{1}{F(x)} + \dfrac{1}{G(x)}}$.

EXTRA CREDIT 5 pts

Sketch the graph of: $y = \left| \dfrac{x^3 - x^2 - 2x + 2}{x-1} \right|$. Consider x values: $-3 \le x \le 3$

I

A) $A(x)=\dfrac{\sqrt{x+3}}{x^2+x}$



$x(x+1)$

D_A $x \geq -3$; $x \neq \{0,-1\}$

B) $B(x)=\sqrt{\frac{1}{x}-x}$

$\sqrt{\frac{1-x^2}{x}}$

$\sqrt{\frac{(1-x)(1+x)}{x}}$

$+\,0\,-\,0\,+\,0\,+$ (−1, 0, 1)

D_B $x \leq -1$ or $0 < x \leq 1$

C) $C(x)=\dfrac{4(x+1)}{(x-1)(x+1)}$

D_C $x \neq \{0, \pm 1\}$

−2 pts

D) $D(x)=\sqrt{-(x-3)(x+1)}$

$-\,0\,+\,0\,-$ (−1, 0, 3)

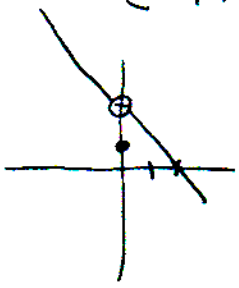
D_D $-1 \leq x \leq 3$

II

A) $A(x)=(\operatorname{Sgn}(x))^2-x+1$

I | II 0 III (3)

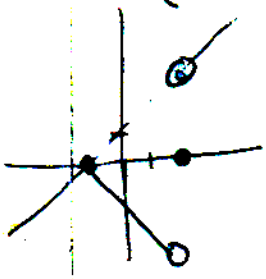
$A(x)=\begin{cases} -x+2 & \text{if } x<0 \\ 1 & \text{if } x=0 \\ -x+2 & \text{if } x>0 \end{cases}$

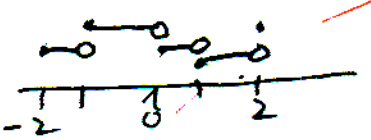


B) $B(x)=|x+1|\operatorname{Sgn}(x-2)$

I | II 0 III | IV 0 V (−1, 2)

$B(x)=\begin{cases} x+1 & \text{if } x<-1 \\ 0 & \text{if } x=-1 \\ -x-1 & \text{if } -1<x<2 \\ 0 & \text{if } x=2 \\ x+1 & \text{if } x>2 \end{cases}$ $\quad -(x+1)(-1)$ $\quad (x+1)(-1)$ $\quad x+1$

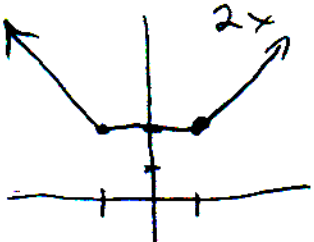




C) $C(x)=|x+1|+|x-1|$

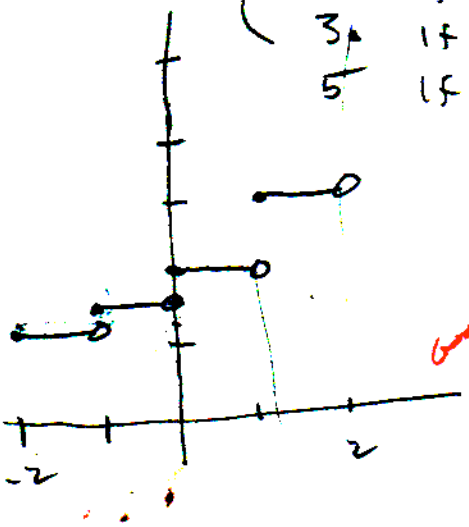
I | II 0 III | IV 0 V (−1, 1)

$C(x)=\begin{cases} -2x & \text{if } x<-1 \\ 2 & \text{if } x=-1 \\ 2 & \text{if } -1<x<1 \\ 2 & \text{if } x=1 \\ 2x & \text{if } x>1 \end{cases}$ $\quad -(x+1)-(x-1)$ $\quad -2x$ $\quad -x+1\,-x+1$ $\quad 2x$



III $A(x)=2^{[x]}+1$

$A(x)=\begin{cases} \frac{5}{4} & \text{if } -2 \leq x<-1 \\ \frac{3}{2} & \text{if } -1 \leq x<0 \\ 2 & \text{if } 0 \leq x<1 \\ 3 & \text{if } 1 \leq x<2 \\ 5 & \text{if } x=2 \end{cases}$



good try −4

IV A(h,k) a=2 d=2

$\frac{y-2}{x-2}=2$

$y-2=2(x-2)$

$(y-2)^2=4(x-2)^2$

$y-2=2(x-2)$

$y-2=\pm 4\frac{\sqrt{5}}{5}$

$y=2\pm\frac{4\sqrt{5}}{5}$ $\left(2-\frac{2\sqrt{5}}{5}, 2-\frac{4\sqrt{5}}{5}\right)$ 4

$4=(x-2)^2+(y-2)^2$

$4=5(x-2)^2$

$\pm\frac{2\sqrt{5}}{5}=x-2$

$x=2\pm\frac{2\sqrt{5}}{5}$ $\left(2+\frac{2\sqrt{5}}{5}, 2+\frac{4\sqrt{5}}{5}\right)$

c) f is 1:1 if $ff^{-1}(x)=x$

$f\left(\frac{2x+3}{5x+2}\right)=\frac{2\left(\frac{2x+3}{5x-2}\right)+3}{5\left(\frac{2x+3}{5x+2}\right)-2}$

lose reflection vertical line test $=\frac{4x+6+15x-6}{10x+15-10x+4}$

$=\frac{19x}{19}$

$=x$ f is 1:1.

d) $H=f(g(x))$

$\frac{2x+3}{x}\neq\frac{2}{5}$

$10x+15=2x$

$8x=-15$

$x=-\frac{15}{8}$

D_H $x\neq\left\{-\frac{15}{8}, 0\right\}$

e) $J(x)=\sqrt{f^{-1}+g^{-1}}$

$\frac{1}{f}+\frac{1}{g}=\frac{6x-2}{2x+3}$

+ 0 0

$-\frac{3}{2}$ 0 $\frac{1}{3}$ $\frac{2}{5}$ $\frac{5}{15}$ $\frac{6}{15}$

D_J $x<-\frac{3}{2}$ or $x\geq\frac{1}{3}$, $x\neq\frac{2}{5}$ 2pts

$\frac{5}{15}$ $\frac{6}{15}$

V $f(x)=\frac{2x+3}{5x-2}$ $G(x)=\frac{2x+3}{x}$

4) $x=\frac{2z+3}{5z-2}$

$5xz-2x=2z+3$

$(5x-2)z=2x+3$

$f^{-1}(x)=z=\frac{2x+3}{5x-2}$

$D_{f^{-1}}$ $x\neq\frac{2}{5}$, R_f $f(x)\neq\frac{2}{5}$

5) $x=\frac{2z+3}{z}$

$xz-2z=3$

$z=\frac{3}{x-2}$

$D_{g^{-1}}$ $x\neq 2$, R_g $g(x)\neq 2$

Extra

x^2-2

$x-1\ \big|\ x^3-x^2-2x+2$

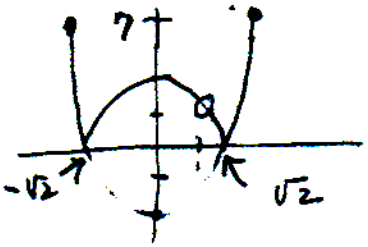
x^3-x^2

$-2x$

$-2x+2$

0

$y=\left|\frac{(x^2-2)(x-1)}{x-1}\right|=|x^2-2|$



Parabola vertex (0,−2)
Neg portion folded up
to become positive,
omit Domain rest $x\neq 1$

I

A) $A(x)=\sqrt{\frac{x^2+5x+6}{x+1}}$

$=\sqrt{\frac{(x+2)(x+3)}{x+1}}$

− 0 + 0 − 0 +

−3 −2 −1 0

D_A $-3\le x\le -2$ or $x>-1$

B) $B(x)=\frac{\sqrt{x^2+5x+6}}{x+1}$

+ 0 − 0 +

−3 −2 −1

D_B $x\le -3$ or $x\ge -2$, $x\neq -1$ except

C) $C(x)=\frac{\sqrt{x-2}}{x^2-4x}$

$=\frac{\sqrt{x-2}}{x(x-4)}$

− 0 + 0 −

0 2 4

D_C $x>2$, $x\neq 4$

D) $D(x)=\sqrt{-x}$

+ 0 −

0

D_x $x\le 0$

II

$A(x)=\frac{3x-2}{5x-4}$

D_A $x\neq \frac{4}{5}$ R_A $A(x)\neq \frac{3}{5}$

$x=\frac{3z-2}{5z-4}$

$5xz-4x=3z-2$

$(5x-3)z=4x-2$

$z=\frac{4x-2}{5x-3}$

$A^{-1}(x)=\frac{4x-2}{5x-3}$

$D_{A^{-1}}$ $x=\frac{3}{5}$!!

B) $B(x)=\frac{3x-2}{6x-4}$

$=\frac{3x-2}{2(3x-2)}$

$B(x)=\frac{1}{2}$

D_B $x\neq \frac{2}{3}$

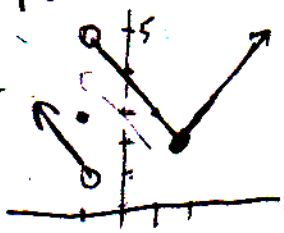
R_B $B(x)=\frac{1}{2}$

C) $C(x)=|2x+1|-3$

D_C $x\in\mathbb{R}$

R_C $C(x)\ge -3$

$|2x+1|$ No smaller than zero ∴ C no smaller than −3.



III

A) $A(x)=\frac{\text{Sgn}(x+2)}{\text{Sgn}(x-3)}$

I 0 II 0 III

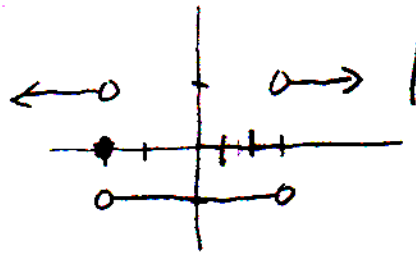
−2 0 3

I $\frac{-1}{-1}=1$

II $\frac{1}{-1}=-1$

III $\frac{1}{1}=1$

$A(x)=\begin{cases}1 & \text{if } x<-2\\ 0 & \text{if } x=-2\\ -1 & \text{if } -2<x<3\\ 1 & \text{if } x>3\end{cases}$



$B(x)=|x-2|+2\,\text{Sgn}(x+1)$

0 0

−1 0 2

I $-(x-2)-2$

$-x+2-2$

$-x$

II $-(x-2)+2$

$-x+4$

III $x-2+2$

x

$B(x)=\begin{cases}-x & \text{if } x<-1\\ 3 & \text{if } x=-1\\ -x+4 & \text{if } -1<x<2\\ x & \text{if } x\ge 2\end{cases}$

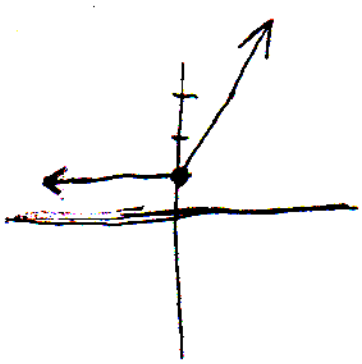
C) $C(x)=x+|x|+1$

0

I $x-x+1$ $x<0$ 1

II $2x+1$

$C(x)=\begin{cases}1 & \text{if } x\le 0\\ 2x+1 & \text{if } x>0\end{cases}$



IV $f(x) = \frac{x-3}{x^2-6x+9}$ $G(x) = \frac{x+3}{x^2+3x+2}$

$= \frac{x+3}{(x+1)(x+2)}$

D_f $x \neq 3$ $\frac{x-3}{(x-3)^2}$

$f(x) = \frac{1}{x-3}$

D_G $x \neq \{-1, -2\}$

A) $A(x) = \frac{G}{f}$ D_A $x \neq \{-2, -1, 3\}$

B) $B(x) = G(f(x))$

D_B $x \neq \{3, 2, \frac{5}{2}\}$

$\frac{1}{x-3} \leq -1$ $\frac{1}{x-3} \leq -2$

$1 \leq -x+3$ $1 \leq -2x+6$

$x \leq 2$ $2x \leq 5$

$x \leq \frac{5}{2}$

C) $C(x) = \sqrt{\frac{1}{G(x)}}$

$-\ 0\ +\ 0\ -\ 0\ +$

$-3 \quad -2 \quad -1 \quad 3$

D_C $-3 < x < -2$ or $x > -1$

D) $D(x) = f(f(x))$

D_D $x \neq \{3, \frac{10}{3}\}$

$\frac{1}{x-3} \leq 3$

$1 \leq 3x-9$

$3x \geq 10$

$x \geq \frac{10}{3}$

V $f(x) = \frac{x}{2x-3}$ $G(x) = \frac{2x-1}{3x-2}$

A) $G\left(\frac{x}{2x-3}\right) = \frac{2\left(\frac{x}{2x-3}\right)-1}{3\left(\frac{x}{2x-3}\right)-2}$

$= \frac{\frac{2x-2x+3}{2x-3}}{\frac{3x-4x+6}{2x-3}}$

$= \frac{3}{6-x}$

B) $f(x) = \frac{x}{2x-3}$

$x = \frac{z}{2z-3}$

$2xz - z - 3x = 0$

$(2x-1)z = 3x$

$z = \frac{3x}{2x-1}$

$f\left(\frac{3x}{2x-1}\right) = \frac{\frac{3x}{2x-1}}{2\left(\frac{3x}{2x-1}\right)-3}$

$= \frac{\frac{3x}{2x-1}}{\frac{6x-6x+3}{2x-1}}$

$= \frac{3x}{3}$

$= x$ fn 1:1

Extra

$x^2 - 4x - 8y + 12 = 0$

$8y = x^2 - 4x + \quad + 12$

$= x^2 - 4x + 4 + 8$

$y = \frac{1}{8}(x-2)^2 + 1$

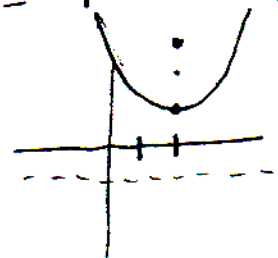
Vertex $(2, 1)$

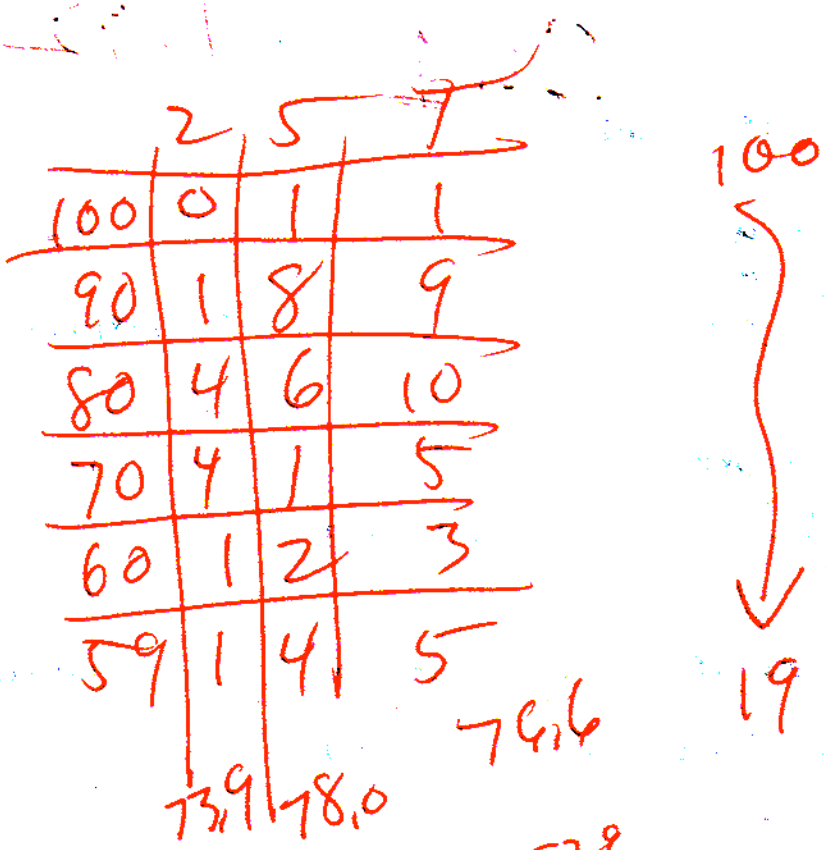
Focus $(2, 3)$

dir $y = -1$

$\frac{1}{4c} = \frac{1}{8}$

$c = 2$





unbelievable!

The bottoms & Give new definition to ignoring all the rules!

I guess, with really weak students you're ~~got~~ to give quizzes ~~—~~ 2 or 3 per week.

The real problem is that this group is "SATisfied"!

Honors Analysis
Copy original problem.
Convince *me* that you understand the concept!

Name ANALYSIS
Per CH 1 Date 1992

Chapter 1 Exam

I State the **Domain ONLY** for each of the following : (5 pts ea)

A) $A(x)=\sqrt{\dfrac{x^2+5x+6}{x+1}}$ B) $B(x)=\dfrac{\sqrt{x^2+5x+6}}{x+1}$ C) $C(x)=\dfrac{\sqrt{x-2}}{x^2-4x}$ D) $D(x)=\sqrt{-x}$

II State the Domain AND Range for each of the following: (5 pts ea)

A) $A(x)=\dfrac{3x-2}{5x-4}$ B) $B(x)=\dfrac{3x-2}{6x-4}$ C) $C(x)=|2x+1|-3$

III Restate as a "piece-meal" function. Sketch a graph of each on a clearly labeled axis. (10 pts ea)

A) $A(x)=\dfrac{\operatorname{sgn}(x+2)}{\operatorname{sgn}(x-3)}$ B) $B(x)=|x-2|+2\operatorname{sgn}(x+1)$ C) $C(x)=x+|x|+1$

IV Given: $F(x)=\dfrac{x-3}{x^2-6x+9}$ and $G(x)=\dfrac{x+3}{x^2+3x+2}$, state **Domain *ONLY*** for: (5 pts ea)

A) $A(x)=\dfrac{G(x)}{F(x)}$ B) $B(x)=G(F(x))$ C) $C(x)=\sqrt{\dfrac{1}{G(x)}}$ D) $D(x)=F(F(x))$

V Given: $F(x)=\dfrac{x}{2x-3}$ and $G(x)=\dfrac{2x-1}{3x-2}$ (15 pts total)

A) Find the **Formula ONLY** for $G(F(x))$.

B) ***Prove*** that F(x) is a 1:1 function.

Extra Credit **5 pts**

Given the parabola in "standard" form: $x^2-4x-8y+12=0$
Determine the coordinates of the vertex, focus and the equation of the directrix.

2'

1993

	2	5	T
100	0	0	0
90	2	5	7
80	10	5	15
70	4	6	10
60	3	1	4
59	3	3	6

99 → 45

1654 | 1576 | 3230

~~1654~~ | 78.8 | 41

~~1576~~ | 78.8

78.8

Excellent exam — students "thought" they knew it.... they never believed -....

No changes in ~~exam~~.

1995

	5	6	T
100	0	0	0
90	0	3	3
80	2	0	2
70	5	1	6
60	3	2	8
59	11	8	19

95 → 11

1209 | 816

21 | 14

57.6 | 58.3 | 57.8

what a bunch of pretenders!!!

You can sure tell I'm not teaching Alg II!

This is about long — if you're not prepared! —

No changes.

I a) $A(x)=\sqrt{9-x^2}$

$-0+0-$ at $-3, 0, 3$

D_A $-3\le x\le 3$

b) $B(x)=\sqrt{1+\frac{1}{x-1}}$

$=\sqrt{\frac{x-1+1}{x-1}}$

$=\sqrt{\frac{x}{x-1}}$

$+0-0+$ at $0, 1$

not -3 factored and -1 wrong

D_B $x\le 0$ or $x>1$

c) $C(x)=\sqrt{25}$

D_C $x\in\mathbb{R}$ ± 5 -3

D) $D(x)=\sqrt{x^2-x-2}$

$=\sqrt{(x-2)(x+1)}$

$+0-0+$ at $-1, 2$

D_D $x\le -1$ or $x\ge 2$

II

A) $A(x)=\operatorname{Sgn}(x-1)-|x+1|$

$+0$ II 0 III at $-1, 1$

I $-1-(-(x+1))$

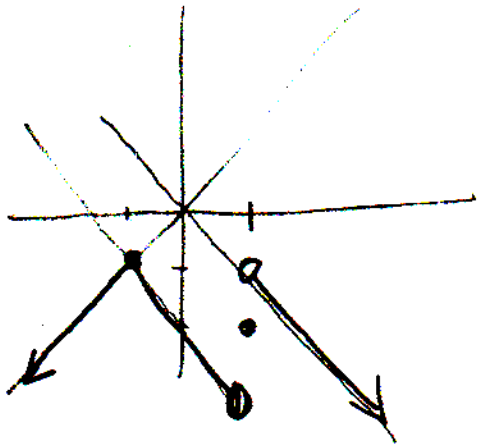
$-1+x+1$

x

II $-1-x-1$

III $1-x-1$

$A(x)=\begin{cases} x & \text{if } x<-1 \\ -1 & \text{if } x=-1 \\ -x-2 & \text{if } -1<x<1 \\ -2 & \text{if } x=1 \\ -x & \text{if } x>1 \end{cases}$



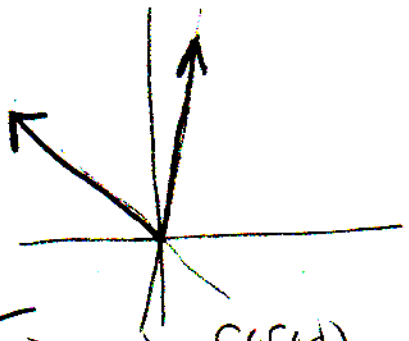
B) $B(x)=x+|x|+x\operatorname{Sgn}(x)$

I 0 II at 0

I $x-x-x=-x$

II $x+x+x=3x$

$B(x)=\begin{cases} -x & \text{if } x\le 0 \\ 3x & \text{if } x>0 \end{cases}$



III $f(x)=\frac{x+4}{(x+4)(x-3)}=\frac{1}{x-3}$ $g(x)=\frac{(4-x)^2}{(x-3)(x+2)}$

D_f $x\neq\{-4,3\}$ D_g $x\neq\{-2,3\}$

a) $A(x)=\frac{f}{g}$

D_A $x\neq\{\pm 4, 3, -2\}$

b) $B(x)=\sqrt{\frac{1}{g}}$

$+0-0+\oplus+$ at $-2, 0, 3, 4$ ← 2 pts

D_B $x<-2$ or $x>3, x\neq 4$ (1 pt)

c) $C(x)=f(f(x))$

$\frac{1}{x}\overset{\text{Set}}{=}-4$ $\frac{1}{x-3}\overset{\text{Set}}{=}3$

$1=-4x+12$ $1=3x-9$

$4x=11$ $3x=10$

$x=\frac{11}{4}$ $x=\frac{10}{3}$

D_C $x\neq\{-2, 3, \frac{11}{4}, \frac{10}{3}\}$

3, 4

Do on next Page

III D $D(x) = \frac{1}{f+g}$

$\frac{1}{x-3} + \frac{(4-x)^2}{(x-3)(x+2)} \overset{\text{set}}{\geq} 0$

$x+2 + 16 - 8x + x^2 = 0$

$x^2 - 7x + 18 = 0$

$\frac{7 \pm \sqrt{49-72}}{}$

$x =$ Reject.

No additional restrictions.

$D_B \; x \neq \{-4, 3, -2\}$

3pts

IV $f(x) = \frac{2x+1}{x-2}$ $g(x) = \frac{1}{x+1}$

a) $h(x) = f(g) = \frac{2\left(\frac{1}{x+1}\right)+1}{\frac{1}{x+1} - 2} = \frac{\frac{2+x+1}{x+1}}{\frac{1-2x-2}{x+1}}$

$h(x) = \frac{x+3}{-2x-1} = -\frac{x+3}{2x+1}$

B) $x = -\frac{z+3}{2z+1}$

$2xz + x = -z - 3$

$(2x+1)z = -x-3$

$z = \frac{-x-3}{2x+1} = h^{-1}(x)$

$h(h^{-1}(x)) = -\frac{\left(\frac{-x-3}{2x+1}\right)+3}{2\left(\frac{-x-3}{2x+1}\right)+1}$

$= -\frac{-x-3+6x+3}{-2x-6+2x+1}$

$= -\frac{5x}{-5} = x$ h is 1:1

V

A (0,2)
B (-3,0)
C (9,1)
D (-6,-9)

4) AB // CD

$M_{AB} = \frac{2-0}{0+3} = \frac{2}{3}$

$M_{CD} = \frac{1+9}{9+6} = \frac{10}{15} = \frac{2}{3}$

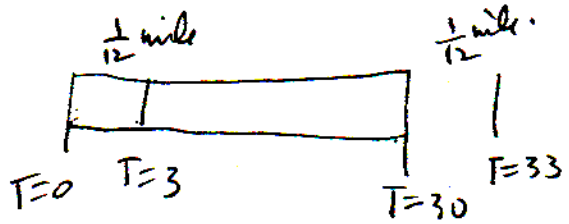
Slopes //

B) Length $\overline{BC}$

$l = \sqrt{(9+3)^2 + 1^2}$

$\sqrt{144+1}$

$\sqrt{145} \approx 5.29$



Times for Engine.

An Additional 30 seconds to pass completely through.

Extra,

$\frac{180 \text{ miles}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{1}{36} \frac{\text{mile}}{\text{sec}}$

A) 3 seconds to pass through entrance $\therefore 3 \cdot \frac{1}{36} = \frac{1}{12}$ mile is length of train.

B) $\frac{5}{30} \cdot \frac{1}{36} = \frac{5}{6}$ of a mile long The Tunnel.

Honors Analysis Name ANL Ch 1

Copy original problem. Per _____ Date 9/4

Convince *me* that you understand the concept!

No calculators.

Chapter 1 Exam

I Determine the domain for each of the following functions. (5 pts ea)

a) $A(x) = \sqrt{9 - x^2}$ b) $B(x) = \sqrt{1 + \frac{1}{x-1}}$ c) $C(x) = \sqrt{25}$ d) $D(x) = \sqrt{x^2 - x - 2}$

II For each of the following, re-define as a "piece-wise" function. Sketch the graph. Specifically state the Range of the function. 10 (15 pts ea)

a) $A(x) = \text{sgn}(x-1) - |x+1|$ b) $B(x) = x + |x| + x\,\text{sgn}(x)$

III Given $f(x) = \frac{x+4}{x^2+x-12}$ and $g(x) = \frac{(4-x)^2}{(x-3)(x+2)}$. Determine the domain for each of the following (Do not actually compute the formula for the new functions.): 10 (5 pts ea)

a) $A(x) = \frac{f(x)}{g(x)}$ b) $B(x) = \sqrt{\frac{1}{g(x)}}$ c) $C(x) = f(f(x))$ d) $D(x) = \sqrt{\frac{1}{f(x) + g(x)}}$ 2 MINUS

IV Given $f(x) = \frac{2x+1}{x-2}$, $g(x) = \frac{1}{x+1}$ and $h(x) = f(g(x))$ (20 pts tot)

a) Determine the formula for $h(x)$. b) Prove $h(x)$ is a 1 : 1 funnction.

V Given: $A(0,2)$, $B(-3,0)$, $C(9,1)$, $D(-6,-9)$, (5 pts ea)

a) Prove the line segment $\overline{AB}$ is parallel to the line segment $\overline{CD}$.

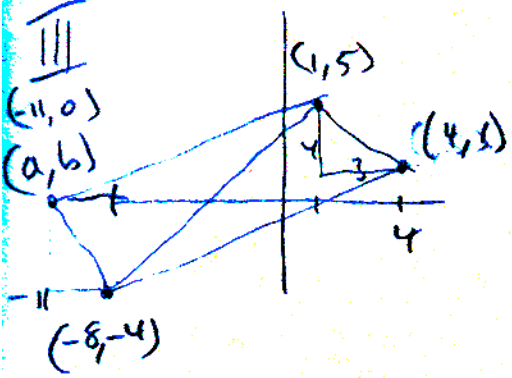
b) Find the length of the line segement $\overline{BC}$

Extra Credit ---------- 5 pts ----------

A train traveling at 100 miles per hour takes three seconds to enter a tunnel and an additional thirty seconds to pass completely through the tunnel.

1. What is the length of the train?
2. How long is the tunnel?
3. Is it true that the more math you know, the harder it is to do arithmetic?

III



4) $(-11,0)$ $(1,5)$ $(-11,0)$ $(-8,-4)$ So opposite sides have same slope ∴ are ||.
$(-8,-4)$ $(4,1)$ $(1,5)$ $(4,0)$

$\frac{0+4}{-11+8}$ $\frac{5-1}{1-4}$ $\frac{5-0}{1+11}$ $\frac{1+4}{4+8}$

$-\frac{4}{3}$ // $-\frac{4}{3}$ $\frac{5}{12}$ // $\frac{5}{12}$

B) $d = \sqrt{(1+8)^2+(5+4)^2}$

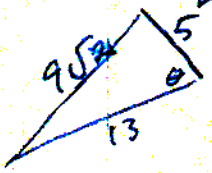
$= 9\sqrt{2}$

$d = \sqrt{(4+11)^2+1}$

$= \sqrt{225+1}$

$\sqrt{226}$ 2.113

Extra. Law of Cos



$162 = 25+169-130\cos\theta$

$\frac{-32}{-130} = \cos\theta = .246$

$\theta = 75°45'$

Area = (base) height = a b sin C

$= 13.5 \sin 75°45'$

$= 63$ sq units

IV $f(x) = \frac{2x+3}{5x-1}$ $g(x) = \frac{2x+3}{7-x}$

a) $x = \frac{2z+3}{5z-1}$

$5xz - x = 2z + 3$

$(5x-2)z = x+3$

$f^{-1}(x) - z = \frac{x+3}{5x-2}$

c) $f(f^{-1}(x)) = \frac{2\left(\frac{x+3}{5x-2}\right)+3}{5\left(\frac{x+3}{5x-2}\right)-1}$

$= \frac{2x+6+15x-6}{5x-2}$

$\frac{5x+15-5x+2}{5x-2}$

$= \frac{17x}{17}$

$= x$

b) $f(f(x)) = \frac{\frac{2x+3}{5x-1}+3}{5\left(\frac{2x+3}{5x-1}\right)-2}$

$= \frac{\frac{2x+3+15x-3}{5x-1}}{\frac{10x+15-10x+2}{5x-1}}$

$= \frac{17x}{17}$

$= x$

e) $\sqrt{\frac{5x-1}{2x+3}+\frac{x-2}{2x+3}}$

$\sqrt{\frac{6x-8}{2x+3}}$

	+	0	−	0	+	
		$-\frac{3}{2}$		$\frac{4}{3}$		
			0			7
				$\frac{1}{5}$		

D_f $x < -\frac{3}{2}$ or $x \geq \frac{4}{3}$; $x \neq 7$

D) $h(x) = \sqrt{\frac{5x-1}{2x+3}+\frac{7-x}{2x+3}}$

$= \sqrt{\frac{4x+6}{2x+3}} = \sqrt{2}$

D_h $x \neq \{-\frac{3}{2}, \frac{1}{5}, 7\}$

This test was a few minutes long
remove 2 parts as indicated.
The reason most people had trouble was
they blindly tried to do what was
done in the HW and on previous years
test ie $y-2 = \frac{3}{4}(x+1)$

$$10 = \sqrt{(x\quad)^2 + (y-2)^2}$$

$$= \sqrt{\qquad + \frac{9}{16}(x+1)^2} \text{ etc.}$$

basically — they didn't read carefully
nor did they think.
make only those changes indicated.

also — many thought they would get ½ pts back — ...

Like last year.
DIDN'T Happen.

	S	G	T
100	0	0	0
90	1	4	5
80	9	5	14
70	8	5	13
60	1	3	4
54	3	3	6
	$\frac{1671}{22}$	$\frac{1503}{20}$	$\frac{3174}{42}$
	75.95	75.15	75.6

97

49

Honors Analysis

Copy original problem.

Convince *me* that you understand the concept!

Name A:N L Ch 1

Per _____ Date 2/16/96

Chapter 1 Exam

I

State the domain for each of the following: (5 pts ea)

a) $A(x) = \sqrt{\dfrac{(2 + x)(3 - x)}{x^2 - 7x + 12}}$

b) $B(x) = \sqrt{\dfrac{3}{5x + 10} + \dfrac{2}{5x - 15}}$

c) $C(x) = \dfrac{1}{1 - \operatorname{sgn}(3x + 7)}$

d) $D(x) = \sqrt{\dfrac{(x + 3)^3 \operatorname{sgn}(x + 2)}{(x - 1)^2}}$

II

For each of the following, re-define as a "piece-wise function. Sketch a graph. Specifically state the range of the function. (10 pts ea)

change the brackets to Layla ( )

a) $P(x) = 2^{\operatorname{sgn}[(x+2)(x-3)]}$

to avoid confusion with Greatest Integer Fn

b) $Q(x) = \dfrac{|x - 1|}{\operatorname{sgn}(x + 1)}$

c) $R(x) = \dfrac{(\operatorname{sgn}(x + 1) + 1)}{(\operatorname{sgn}(x - 1) - 1)}$

III

Given coordinates: $(1, 5)$, $(4, 1)$, $(-8, -4)$, and (a, b). (25 pts tot)

a) Determine (a, b) so the figure formed by connecting the points in order is a parallelogram. Be sure to prove that you have a parallelogram (and not a trapazoid, for example). Explain.

4 Triples

b) Determine the lengths of the diagonals of the parallelogram. Be very sure to show your work.

IV

Given $f(x) = \dfrac{2x + 3}{5x - 1}$ and $g(x) = \dfrac{2x + 3}{7 - x}$ ~~(5 pts ea)~~

25 pts tot.

a) Find $f^{-1}(x)$

b) Find $f\left(f^{-1}(x)\right)$

~~c) Find $f^{-1}\left(f(x)\right)$~~

d) Find the domain of $h(x) = \sqrt{\dfrac{1}{f(x)} + \dfrac{1}{g(x)}}$

~~e) Find the domain of $j(x) = \sqrt{\dfrac{1}{f(x)} - \dfrac{1}{g(x)}}$~~

Extra Credit ---------- 5 pts ----------

Find the area of the parallelogram in part III.

1997

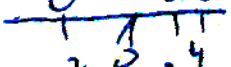
100	0
90	3
80	7
70	7
60	7
59	1

$\frac{1908}{25}$ 76.32

I

4) $A(x) = \sqrt{\frac{(2+x)(3-x)}{x^2-7x+2}}$ $(x-3)(x-4)$

$= \sqrt{\frac{-(2+x)}{x-4}}$



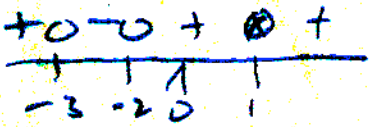
D_4 $-2 \le x < 4$, $x \neq 3$

c) $C(x) = \frac{1}{1-\operatorname{Sgn}(3x+7)}$

$\operatorname{Sgn}(3x+7) = 1$ if $x > -\frac{7}{3}$

D_C $x \le -\frac{7}{3}$

D) $D(x) = \sqrt{\frac{(x+3)^3 \operatorname{Sgn}(x+2)}{(x-1)^2}}$

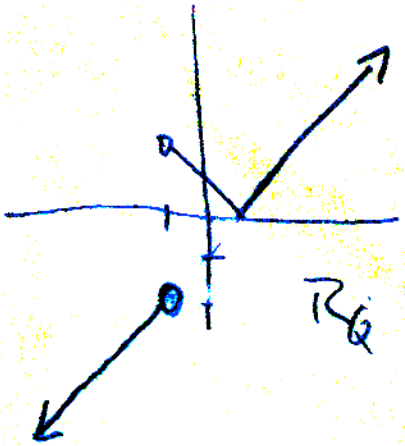


D_D $x \le -3$ or $x \ge -2$; $x \neq 1$

II

b) $Q(x) = \frac{|x-1|}{\operatorname{Sgn}(x+1)}$

$$Q(x) = \begin{cases} x-1 & \text{if } x < -1 \\ -x+1 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x \ge 1 \end{cases}$$

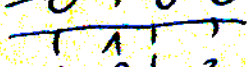


R_Q $Q(x) < -2$ or $Q(x) \ge 0$

6) $B(x) = \sqrt{\frac{3}{5x+10} + \frac{2}{5x-15}}$ $\frac{3}{5(x+4)}$ $\frac{2}{5(x-3)}$ NOT added −3

$= \sqrt{\frac{3(x-3)+2(x+2)}{5(x+2)(x-3)}}$

$= \frac{3x-9+2x+4}{5x-5}$

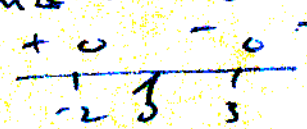


$\sqrt{\frac{5(x-1)}{5(x+2)(x-3)}}$

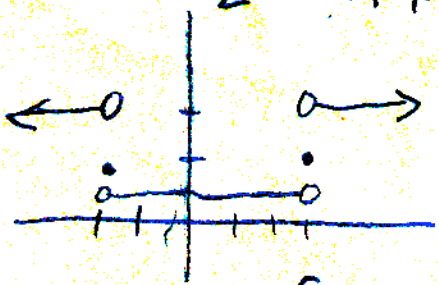
D_B $-2 < x \le 1$ or $x > 3$

II

a) $P(x) = 2^{\operatorname{Sgn}[(x+2)(x-3)]}$

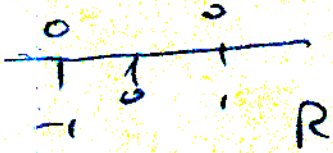


$$P(x) = \begin{cases} 2 & \text{if } x < -2 \\ 1 & \text{if } x = -2 \\ \frac{1}{2} & \text{if } -2 < x < 3 \\ 1 & \text{if } x \ge 3 \\ 2 & \text{if } x > 3 \end{cases}$$

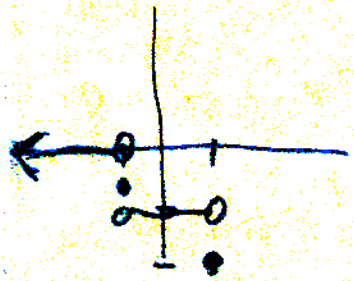


R_P $P(x) \in \{\frac{1}{2}, 1, 2\}$

c) $R(x) = \frac{\operatorname{Sgn}(x+1)+1}{\operatorname{Sgn}(x-1)-1}$

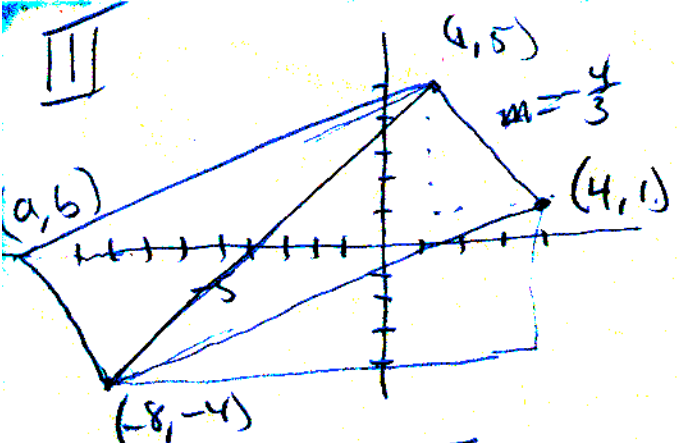


$$R(x) = \begin{cases} 0 & \text{if } x < -1 \\ -\frac{1}{2} & \text{if } x = -1 \\ -1 & \text{if } -1 < x < 1 \\ -2 & \text{if } x > 1 \end{cases}$$



$R_{R(x)}$ $R(x) \in \{-2, -1, -\frac{1}{2}, 0\}$

II



A) $\frac{5-1}{4-4} = \frac{4}{3}$ $\quad \frac{4+4}{4+8} = \frac{5}{12}$

$\frac{b+4}{a+8} = -\frac{4}{3}$ $\quad \frac{b-5}{a-1} = \frac{5}{12}$

$b+4 = -\frac{4}{3}(a+8)$ $\quad b = \frac{5}{12}(a-1)+5$

$\left(-\frac{4}{3}(a+8)-4 = \frac{5}{12}(a-1)+5\right)12$

$-16a-128-48 = 5a-5+60$

$-176 = 21a+55$

$-231 = 21a$

$a = -11$

$\frac{b-5}{-12} = \frac{5}{12}$

$b-5 = -5$

$b = 0$

b) $d = \sqrt{(1+8)^2+(5+4)^2}$

$= \sqrt{81+81} = 9\sqrt{2}$

$d = \sqrt{(4+11)^2+1}$

$= \sqrt{225+1}$

$= \sqrt{226}$ $\quad 2.113$

IV

$f(x) = \frac{2x+3}{5x-1}$

$D_f\ x \neq \frac{1}{5}$

$g(x) = \frac{2x+3}{7-x}$

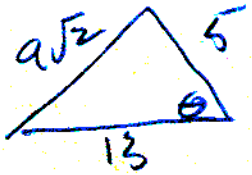
$D_g\ x \neq 7$

a) $x = \frac{2z+3}{5z-1}$

$5xz - x = 2z+3$

$(5x-2)z = x+3$

$z = \frac{x+3}{5x-2}$



Extra.

Law of cos

$a^2 = b^2+c^2-2bc\cos A$

$16z = 25+169-130\cos\theta$

$\frac{-32}{-130} = \cos\theta = .246$

$\theta = 75^\circ 45'$

$2\cdot\frac{1}{2}bh = bc\sin A$

$= 5\cdot 13\sin 75^\circ 45'$

$= 63$ sq units.

b) $f\left(\frac{x+3}{5x-2}\right) = \frac{2\left(\frac{x+3}{5x-2}\right)+3}{5\left(\frac{x+3}{5x-2}\right)-1}$

$= \frac{2x+6+15x-6}{5x+15-5x+2}$

$= \frac{17x}{17}$

$= x$

c) D_h

d) $h = \sqrt{\frac{5x-1}{2x+3} + \frac{2-x}{2x+3}}$

$= \sqrt{\frac{4x+6}{2x+3}}$

$h(x) = \sqrt{2}$

e) $D_h\ x \neq \left\{-\frac{3}{2}, \frac{1}{5}, 7\right\}$

B

2/13/98

100	0
90	3
80	6
70	5
68	5
59	3

94

↓

1625
22

No changes. Super thesis

73.86

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

Name ANL Ch 1

Per _____ Date 9/8

Chapter 1 Exam

I State the domain for each of the following: (5 pts ea)

a) $A(x) = \sqrt{\dfrac{(2 + x)(3 - x)}{x^2 - 7x + 12}}$

b) $B(x) = \sqrt{\dfrac{3}{5x + 10} + \dfrac{2}{5x - 15}}$

c) $C(x) = \dfrac{1}{1 - \operatorname{sgn}(3x + 7)}$

d) $D(x) = \sqrt{\dfrac{(x + 3)^3 \operatorname{sgn}(x + 2)}{(x - 1)^2}}$

II For each of the following, re-define as a "piece-wise" function. Sketch a graph. Specifically state the range of the function. (10 pts ea)

a) $P(x) = 2^{\operatorname{sgn}((x+2)(x-3))}$

b) $Q(x) = \dfrac{|x - 1|}{\operatorname{sgn}(x + 1)}$

c) $R(x) = \dfrac{(\operatorname{sgn}(x + 1) + 1)}{(\operatorname{sgn}(x - 1) - 1)}$

III Given coordinates: $(1, 5)$, $(4, 1)$, $(-8, -4)$, and (a, b). (25 pts tot)

a) Determine (a, b) so the figure formed by ***connecting the points in order*** is a parallelogram. Be sure to prove that you have a parallelogram (and not a trapazoid, for example). Explain.

b) Determine the lengths of the diagonals of the parallelogram. Be very sure to show your work.

IV Given $f(x) = \dfrac{2x + 3}{5x - 1}$ and $g(x) = \dfrac{2x + 3}{7 - x}$ (25 pts tot)

a) Find $f^{-1}(x)$

b) Find $f\left(f^{-1}(x)\right)$

c) Find the domain of $h(x) = \sqrt{\dfrac{1}{f(x)} + \dfrac{1}{g(x)}}$

Extra Credit ---------- 5 pts ----------

Find the area of the parallelogram in part III.

I

A) $A(x)=\sqrt{\frac{x^2+x-2}{x^2-6x+9}}$

$=\sqrt{\frac{(x-1)(x+2)}{(x-3)^2}}$

+ 0 − 0 + ⊗ +

−2 0 1 3

D_A $x\le -2$ or $x\ge 1$; $x\neq 3$

B) $B(x)=\sqrt{\frac{x^2-3x}{x^2-4x+3}}$

$\sqrt{\frac{x(x-3)}{(x-1)(x-3)}}$

+ 0 − 0 + ⊗ +

0 1 3

D_B $x\le 0$ or $x\ge 1$; $x\neq 3$

C) $C(x)=\sqrt{\frac{16}{9}}$

D_C $x\in\mathbb{R}$

D) $D(x)=\frac{\sqrt{x^2-2x-2}}{(x+6)^2}$

$x=\frac{2\pm\sqrt{4+8}}{2}$

$=\frac{2\pm 2\sqrt{3}}{2}$

$1\pm\sqrt{3}$

+ 0 − 0 +

$1-\sqrt{3}$ 0 $1+\sqrt{3}$

D_D $x\le 1-\sqrt{3}$; $x\neq -6$ OR $x\ge 1+\sqrt{3}$

NO $x=\frac{-6\pm\sqrt{}}{2a}$ (−4)

II

4) $A(x)=\operatorname{Sgn}(x-1)-|x+1|$

I 0 II 0 III

−1 1

I $-1+(x+1)$

x

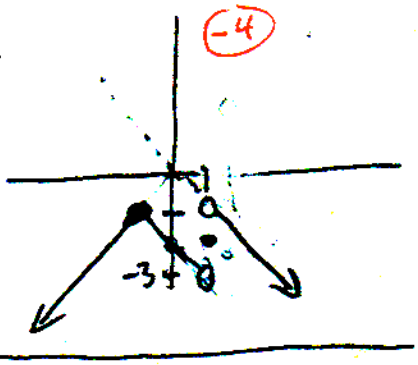
II $-1-x-1$

$-x-2$

III $1-x-1$

$-x$

$$A(x)=\begin{cases} x & \text{if } x\le -1 \\ -x-2 & \text{if } -1<x<1 \\ -2 & \text{if } x=1 \\ -x & \text{if } x>1 \end{cases}$$



B) $B(x)=(|x-1|)^{\operatorname{Sgn}|x|}$

I 0 II 0 III

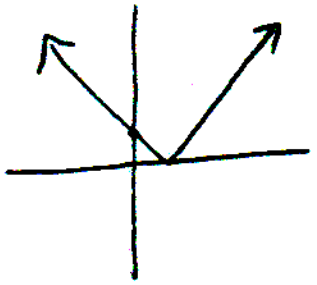
0 1

I $(-(x-1))^1=-x+1$

II $(-(x-1))^1=-x+1$

III $x-1$

$$B(x)=\begin{cases} -x+1 & \text{if } x\le 1 \\ x-1 & \text{if } x>1 \end{cases}$$



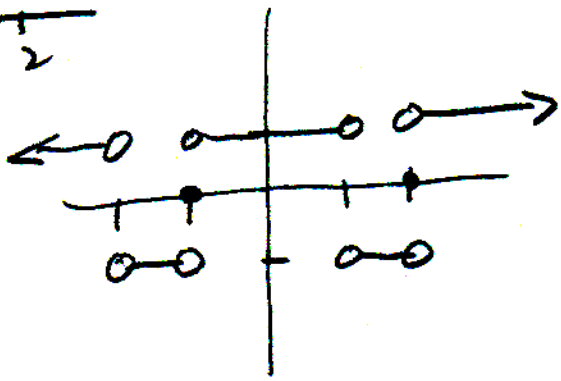
C) $C(x)=\operatorname{Sgn}\left(\frac{x^2-x-2}{x^2+x-2}\right)$

$\frac{(x-2)(x+1)}{(x+2)(x-1)}$

+ 0 − 0 + 0 − 0 +

−2 −1 0 1 2

$$C(x)=\begin{cases} 1 & \text{if } x>2 \\ 1 & \text{if } x<-2 \\ -1 & \text{if } -2<x\le -1 \\ 0 & \text{if } x=-1 \\ 1 & \text{if } -1<x<1 \\ -1 & \text{if } 1<x<2 \\ 0 & \text{if } x=2 \end{cases}$$



III $f(x)=\frac{2x+1}{x-2}$ $G(x)=\frac{x+2}{x^2+x-2}=\frac{x+2}{(x+2)(x-1)}=\frac{1}{x-1}$

D_f $x\neq 2$ D_g $x\neq\{-2,1\}$

A) $A(x)=\frac{G}{f}$

$D_A = x\neq\{-2,-\frac{1}{2},2,1\}$

B) $B(x)=\sqrt{f\cdot g}$

$-\infty\ -0\ +\ 0\ -0\ +$ on $-2,-\frac{1}{2},0,1,2$

D_B $-\frac{1}{2}\leq x<1$ or $x>2$

C) $C(x)=G(f(x))$

D_C $x\neq\{2,\frac{3}{4},-3\}$

$\frac{2x+1}{x-2}\overset{SP}{=}-2$ $2x+1=-2x+4$ $4x=3$ $x=\frac{3}{4}$

$\frac{2x+1}{x-2}\overset{SP}{=}1$ $2x+1=x-2$ $x=-3$

IV $f(x)=\frac{3x+1}{x-3}$

a) D_f $x\neq 3$ R_f $f(x)\neq 3$

$x=\frac{3z+1}{z-3}$ $xz-3x=3z+1$ $(x-3)z=3x+1$ $f^{-1}(x)=z=\frac{3x+1}{x-3}$

B) $f(f^{-1}(x))=\frac{3\left(\frac{3x+1}{x-3}\right)+1}{\frac{3x+1}{x-3}-3}=\frac{9x+3+x-3}{3x+1-3x+9}$

Doesn't show this −6

$\frac{10x}{10}=x$ f is 1:1

V (x,x) $(1,4)$ $(9,0)$ $\sqrt{(x-1)^2+(x-4)^2}\overset{SP}{=}\sqrt{(x-9)^2+x^2}$

$x^2-2x+1+x^2-8x+16=x^2-18x+81+x^2$

$8x+17=81$ $8x=64$ $x=8$ the point is (8,8)

Flowchart −8

EXTRA $(10,-2)$ $(4,6)$ $(-11,-2)$

$y=\sqrt{(x-2)^2}+4$ $(x-2)\vee 17=$ $\sqrt{100}=10$

$(-11,-2)$ $10,-2$

NOT isosceles since $(4,6)$ NOT between $(-11,-2)$ $(10,-2)$

$(10-4)^2+(-2-6)^2=100$ $36+64=100$

$(4+11)^2+(6+2)^2$ 15^2+8^2 $225+64$ $289=17$

100 289 $389\neq 441$ ∴ NOT RT Δ.

$\frac{6+2}{4-10}=\frac{8}{-6}=-\frac{4}{3}$ $\frac{6+2}{4+11}=\frac{8}{15}\neq\frac{-3}{4}$ ∴ NOT RT Δ.

2/12/99

	H	S	T
100	0	0	
90	0	2	
80	5	3	
70	5	12	
60	7	5	
59	3	3	

96

36

	1381	1802	3183
	20	25	
	69.0	72.1	70.7

tough extm. especially for students with poor alg skills.

only change is to put No calculators on the extm. I didn't allow calculator.

Honors Analysis
Copy original problem.
Convince *me* that **you** understand the concept!
No Calculators.

Name _____

Per _____ Date _____

Chapter 1 Make - Up Exam

I State the Domain ONLY for each of the following: (5 pts ea)

A) $A(x) = \sqrt{\dfrac{x^2 + x - 2}{x^2 - 6x + 9}}$ B) $B(x) = \sqrt{\dfrac{x^2 - 3x}{x^2 - 4x + 3}}$ C) $C(x) = \sqrt{\dfrac{16}{9}}$

D) $D(x) = \dfrac{\sqrt{x^2 - 2x - 2}}{(x+6)^2}$

II Re-define each as a "piece-meal" function. Draw the graph. (10 pts ea)

A) $A(x) = \operatorname{sgn}(x-1) - |x+1|$ B) $B(x) = \left(|x-1|\right)^{\operatorname{sgn}|x|}$ C) $C(x) = \operatorname{sgn}\left(\dfrac{x^2 - x - 2}{x^2 + x - 2}\right)$

III Given: $F(x) = \dfrac{2x+1}{x-2}$ and $G(x) = \dfrac{x+2}{x^2+x-2}$.

$A(x) = \dfrac{G(x)}{F(x)}$, $B(x) = \sqrt{F(x)G(x)}$, $C(x) = G(F(x))$ (25 pts tot)

A) Find the domain only for $A(x)$.

B) Find the domain only for $B(x)$.

C) Find the domain only for $C(x)$.

IV Given: $F(x) = \dfrac{3x+1}{x-3}$. (15 pts tot)

A) Find the domain and range. B) Prove F is a 1:1 function.

V Given the point A (2, 2). The point B is 2 units from point A and the line segment $\overline{AB}$ has slope 2.
Find the coordinates of point B. (10 pts)

Honors Analysis
Copy original problem.
Convince *me* that you understand the concept!

Name ANL Ch1

Per _____ Date 9/9

NO calculators

Chapter 1 Exam

I State the Domain ONLY for each of the following: (5 pts ea)

A) $A(x)=\sqrt{\dfrac{x^2+x-2}{x^2-6x+9}}$ B) $B(x)=\sqrt{\dfrac{x^2-3x}{x^2-4x+3}}$ C) $C(x)=\sqrt{\dfrac{16}{9}}$

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$A(x)=\dfrac{G(x)}{F(x)}$, $B(x)=\sqrt{F(x)G(x)}$, $C(x)=G(F(x))$

A) Find the domain only for $A(x)$.

B) Find the domain only for $B(x)$.

C) Find the domain only for $C(x)$.

IV Given: $F(x)=\dfrac{3x+1}{x-3}$. (15 pts tot)

A) Find the domain and range. B) Prove F is a 1:1 function.

V Find the coordinates of the point whose ordinate is equal to its abscissa and which is equidistant from $(1,4)$ and $(9,0)$. (10 pts)

Extra Credit ---------- **5 pts** ----------

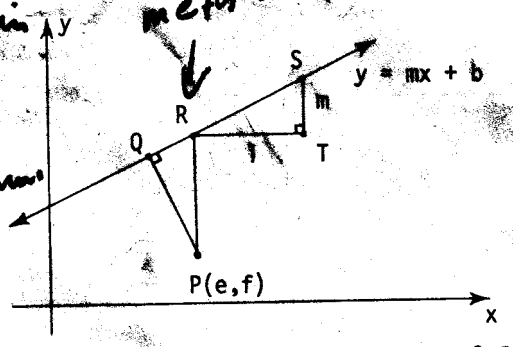
Given the points: $(10,-2)$, $(4,6)$, and $(-11,-2)$. Is the triangle isosceles? Is it a right triangle? Be very clear in your method.

Point to Line, Short and Sweet

Hal Andersen

A question that invariably comes up at a cocktail party is that of finding the distance between a point and a line in a plane. Cocktail party attendees are not satisfied with just an answer; they demand derivation of a solution. The time-honored derivation* is too lengthy to be of interest to sophisticates at a cocktail party, so the next time the question arises present the following:

Consider the diagram where $y = mx + b$ is the given line and $P = (e,f)$ is the given point.



RT = 1;

ST = m;

PR = $|me + b - f|$;

RS = $\sqrt{m^2 + 1}$,

and

$\Delta PQR \sim \Delta RTS$.

Therefore, $\frac{PQ}{PR} = \frac{RT}{RS}$; $PQ = \frac{PR \times RT}{RS} = \frac{|me + b - f|}{\sqrt{m^2 + 1}}$.

EXAMPLE: Find the distance between $y = \frac{1}{3}x + 2$ and $(3,7)$

ANSWER: $PQ = \frac{|3(1/3) + 2 - 7|}{\sqrt{\frac{1}{9} + 1}} = \frac{2}{\sqrt{\frac{10}{9}}} = \frac{6}{\sqrt{10}}$ or $\frac{3}{5}\sqrt{10}$.

* This argument was discovered by Mike Connolly when he was a student at San Carlos High School.

Hal Andersen, Department of Mathematics, Santa Rosa Junior College, 1501 Mendocino Avenue, Santa Rosa, CA 95401.

I

A) $A(x) = \dfrac{x\sqrt{x}}{x^2-5x+6}$

$(x-2)(x-3)$

D_A $x \geq 0$, $x \neq \{2, 3\}$

B) $B(x) = \sqrt{\dfrac{x+3}{x^2+x}}$

− 0 + 0 − 0 +

-3 -1 0

D_B $-3 \leq x < -1$ OR $x > 0$

C) $C(x) = \sqrt{x^2+8x+12}$

+ 0 − 0 +

-6 -2 0

$(x+2)(x+6) > 0$

D_C $x \geq -2$ or $x \leq -6$

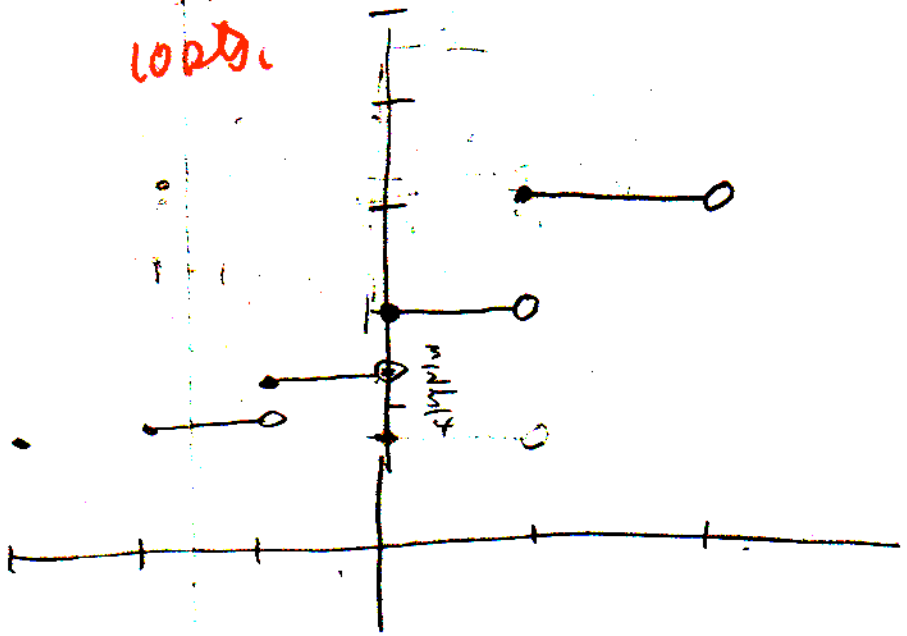
D) $D(x) = \sqrt{25}$

D_D $x \in \mathbb{R}$.

II

$A(x) = 2^{[x]} + 1$ $[-3, 2]$ $(2, 5)$

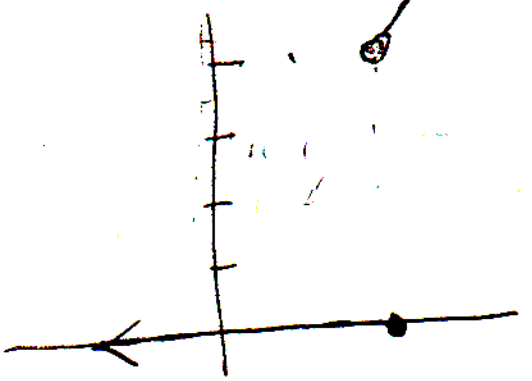
100%.



III

$A(x) = x + x\,\mathrm{sgn}(x-2)$

$A(x) = \begin{cases} 0 & x \leq 2 \\ 2 & x = 2 \\ 2x & x > 2 \end{cases}$ $(3, 9)$

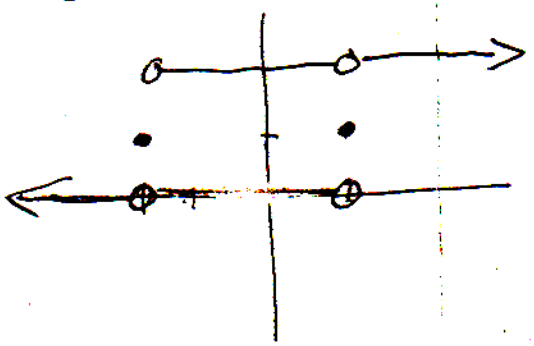


III

B) $B(x) = (\mathrm{sgn}(x-1))^2 + \mathrm{sgn}(x+2)$

1 -1

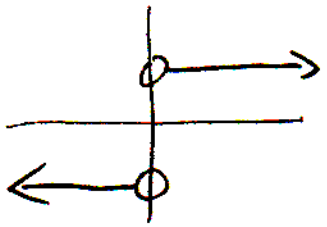
-2 1



$\begin{cases} 0 & \text{if } x < -2 \\ 1 & \text{if } x = -2 \\ 0 & \text{if } -2 < x < 1 \\ 1 & \text{if } x = +1 \\ 2 & \text{if } x > 1 \end{cases}$

III $C(x) = \frac{x}{|x|}$

$$\begin{cases} -1 & \text{if } x<0 \\ 1 & \text{if } x>0 \end{cases}$$



IV $f(x) = \frac{2x+3}{x}$ $\quad g(x) = \frac{2x+3}{5x-2}$

$D_f \; x \neq 0$ $\quad \therefore D_g \; x \neq \frac{2}{5}$

4) $A(x) = \frac{f}{g}$

$D_A \; x \neq \{0, \frac{2}{5}, -\frac{3}{2}\}$

B) $B(x) = G(f(x))$

$D_B \; x \neq \{0, -\frac{15}{8}\}$

c) $C(x) = \frac{1}{\frac{1}{f} + \frac{1}{g}}$

$D_C \; x \neq \{0, \frac{2}{5}, -\frac{3}{2}, \frac{1}{3}\}$

$\frac{2x+3}{x} = \frac{2}{5}$

$10x+15 = 2x$

$8x = -15$

$x = -\frac{15}{8}$

$\frac{x}{2x+3} + \frac{5x-2}{2x+3} = 0$

$\frac{6x-2}{2x+3} = 0$

$x = 3$

V $G(x) = \frac{2x+3}{5x-2}$

$x = \frac{2z+3}{5z-2}$

$5xz - 2x = 2z + 3$

$(5x-2)z = 2x+3$

$z = \frac{2x+3}{5x-2}$

$G(G^{-1}(x)) = \frac{2(\frac{2x+3}{5x-2})+3}{5(\frac{2x+3}{5x-2})-2}$

$= \frac{4x+6+15x-6}{10x+15-10x+4}$

$= \frac{19x}{19} = x$

$\therefore$ G is 1:1

VI $f(x) = (x+2)(x-3)$

$f(2) = 4 \cdot -1 = -4$

$(2, -4) \quad m = 3$

$y + 4 = 3(x-2)$

$y = 3x - 10$

x intercept when $y = 0$

$x = \frac{10}{3}$

$\therefore$ coord $(\frac{10}{3}, 0)$

ANALYSIS CHAPTER 1 MAKE UP NAME Analysis

Per Ch 1

I State the DOMAIN ONLY for each of the following. (5 pts ea)

A) $A(x) = \dfrac{x\sqrt{x}}{x^2 - 5x + 6}$ B) $B(x) = \sqrt{\dfrac{x+3}{x^2+x}}$ C) $C(x) = \sqrt{x^2 + 8x + 12}$ D) $D(x) = \sqrt{25}$

II Graph: $A(x) = 2^{[x]} + 1$ given D_A $x \in [-3, 2]$ where $[x]$ means "greatest integer of x" (10 pts)

III Re-define each as a "piece-meal function". Draw the graph. (10 pts ea)

A) $A(x) = x + x\,Sgn(x-2)$ B) $B(x) = (Sgn(x-1))^2 + Sgn(x+2)$ C) $C(x) = \dfrac{x}{|x|}$

IV Given $f(x) = \dfrac{2x+3}{x}$ $g(x) = \dfrac{2x+3}{5x-2}$ (10 pts ea)

State the domain only for :

A) $A(x) = \dfrac{f(x)}{g(x)}$ B) $B(x) = g[f(x)]$ C) $C(x) = \dfrac{1}{\frac{1}{f(x)} + \frac{1}{g(x)}}$

V Prove the function g(x) mentioned in part IV above is or is not a 1 : 1 function. (5 pts)

VI A line is tangent to $f(x) = (x+2)(x-3)$ at $x = 2$. The slope of the tangent line is 3. What are the coordinates of the x-intercept of that tangent line? (5 pts)